Optimal merger standards for potential competition: the effect of ex ante investment incentives*

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Abstract

This paper presents a model of merger review of potential competition starting from the entrant's investment decision through the government's merger review process to analyze how merger review ought to respond to the problem of a dominant firm buying a potential competitor given uncertainty about whether the entrant will be a competitor or a complement. The potential entrant decides whether to invest at all, and if so, whether to invest in a substitute or complement product to the dominant firm. The investment may fail or may succeed in creating a new product. If the investment succeeds, the dominant firm then initiates an acquisition, which the competition authority reviews receiving only an imperfect signal of whether the merger is between substitutes or complements. The paper shows that considering the incentive to innovate can sometimes make it optimal to loosen merger review standards, but, surprisingly, can also sometimes make it optimal to make merger review even more strict than the ex post optimal standard–often in order to induce the entrant to choose the complement investment rather than the substitute one. This occurs under a wider range of parameter values under a total welfare standard than under a consumer welfare standard.

1 Introduction

Mergers of established firms with potential competitors have come under increasing scrutiny. Government approvals of Facebook's acquisitions of What'sApp and Instagram have been criticized as preventing a potential rival to Facebook from emerging in the social networking space. Others have criticized the approval of the Google-Double Click merger as preventing future competition in search advertising. Facebook and Google, on the other hand, argue that these

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mergers were merely mergers of complements that improved efficiency in the market and that none of these smaller firms would have emerged as successful competitors on their own. That said, the concern over dominant firms using acquisitions to of nascent competitors to preserve their dominant position has led to calls for much stricter review of mergers between dominant firms and smaller firms that could be future competitors. This is reflected both in political circles, such as in the recent US House of Representatives report "Investigation of Competition in Digital Markets" (2020), and among leading academics, such as Scott-Morton et al. (2019), Furman et al. (2019) and Cremer, de Montjoye, and Schweitzer (2019).

Some have pushed back against these arguments, claiming that stricter merger review is not the best approach to controlling the market power of dominant firms. Cabral (2020), for example, argues that the prospect of merging with dominant firms is an important component of innovation incentives for new entrants. He claims that in high-tech markets in particular, it can be very hard to tell whether a new entrant would become an effective potential competitor. Thus, strict merger policy would have many false positives and greatly diminish the incentive to innovate. Moreover, it is not just academics who are concerned about the effect of merger policy on innovation and entry. Venture capitalists have expressed concern to Congress that stricter merger review will both reduce the incentive to launch new firms and for venture capitalists to fund them (Nakache 2019). It is not surprising that merger standards would affect venture capital funding since acquisition are the most common way for venture capital funds to realize a return on their investments (Dushnitsky and Sokol 2021).

This paper develops a formal model of innovation incentives and merger review that explicitly takes into account both the ex ante incentive effects of merger standards and the ex post welfare effects under conditions of uncertainty. In addition, the model not only considers the incentive to innovate versus not innovating, it also considers the effect of merger standards on the potential entrant's type of innovation (attempting to develop either a substitute or complement to the dominant firm). In the model, a potential entrant observes its probability of successfully developing a substitute product and its probability of successfully developing a complement product. Knowing the merger review standard, and thus the probability that a merger with the incumbent will be approved for each type of product, the potential entrant then decides whether to attempt to develop either the substitute, the complement (both at a fixed cost) or nothing at all. Observing entrant's product type (if innovation was attempted), the incumbent and entrant attempt to merge at a price that splits the surplus from the merger. The competition authority then receives an imperfect signal of the type of product the entrant is developing and blocks the merger if and only if this signal is above its standard (higher signals are more likely to come from substitute products).

Thus, in the model, the government can commit to its merger review threshold. In so doing, it chooses the threshold to optimize over three dimensions: the incentive to invest at all, the incentive to choose the socially optimal type of investment, and ex post efficiency. While laxer merger standards encourage all

types of investment, unless those standards are very strict, laxer standards encourage the substitute investment more than the complement investment. Thus, it is not obviously the case that overall investment incentives are improved by laxer merger standards both because investment incentives for substitutes can be excessive (under a total welfare standard, although this is less likely under a consumer welfare standard), and because there can be an excessive incentive for the entrant to choose the substitute product over the complement product (which is also worse under a total welfare standard than under a consumer welfare standard). As a result, considering ex ante investment under a total welfare standard could lead to either a stricter or more lenient optimal merger standard than the one that maximizes ex post total welfare. This can also be the case using a consumer welfare standard. In our numerical simulations, we find that there are many cases in which the ex ante optimal merger standard is more lenient than the ex post optimal one and many cases in which the reverse is true under both welfare standards, but the fraction of cases in which the ex ante standard is more lenient is larger under a consumer welfare standard than under a total welfare standard. Moreover, while we find that the ex ante optimal standard generates a probability of approval of a substitute merger that is at least 10 percentage points lower t from the ex post optimal standard in almost 3% of the cases and is at least 10 percentage points higher than under the ex post optimal standard in about 4% of the cases under the total welfare standard. Under the consumer welfare standard a difference of this magnitude only occurs in the direction of greater leniency under the ex ante optimal standard (in about 7.5% of the cases).

The numerical simulations also show that when the parameters suggest that entry is less likely to be profitable, then the ex ante standard tends to be more lenient relative to the ex post one under both welfare standards. But, for parameter values in which entry is likely to be quite profitable regardless, then changes in the parameter values that tend to make the substitute investment more profitable also make the ex ante optimal standard stricter relative to the ex post optimal one, confirming the intuition that optimal merger standards need to take into account the type of investment they incentivize as well as whether any investment is profitable at all. This effect is more pronounced for the total welfare standard because the type of entry typically matters more for total welfare than for consumer welfare.

Rasmusen (1988) was one of the first papers to model how the prospect of a merger can be an inducement to entry. Segal and Whinston (2007) examined how antirust should respond to innovative industries. Phillips and Zhdanov (2013) develop a model with similar results and empirically test it. Mason and Weeds (2013) add merger review to an entry model to suggest that merger policy should respond to this by allowing acquisitions of new entrants whose profitability is sufficiently low.

In the last few years, there have been a number of contributions to this literature. Cabral (2018) presents a dynamic innovation model in which he examines the effect of allowing the dominant firm to purchase the innovation of the fringe firm has on incentives for both incremental and radical innovation.

He finds that allowing mergers (technology transfers in his model) increases the incentive for incremental innovation but decreases the incentive for radical innovation (in which the fringe firm becomes the dominant firm). He does not distinguish between incentives for substitute versus complement innovation. Furthermore, because his paper is not primarily about merger review, he does not explicitly model optimal merger standards nor consider ex post efficiency issues. Motta and Peitz (2021) also develop a model that shows how mergers can improve ex ante investments due to the elimination of financial constraints but could also lead to reduced competition if the investment would occur without a merger.

Bryan and Hovenkamp (2020) consider a duopoly model with a leading firm and a laggard who are vertically differentiated in the expected way. A potential innovator (the startup) is the third player that decides to innovate in a technology that helps the leader or the laggard. They show that in the absence of restrictions on mergers, the innovator will be biased towards innovations that complement the leader's product. They then discuss how various antitrust interventions might alleviate this problem. In this model, all innovations are complements, and the role of merger review is not to prevent the mergers of substitutes. Thus, they do not focus on optimal merger standards under conditions of uncertainty about the type of merger being proposed.

Kamepalli et al. (2020) model the effect of mergers on innovation incentives in a model with network externalities and switching costs. They find that the possibility of the incumbent buying the entrant deters consumers from trying the entrant's product because they might bear switching costs for a product that soon won't exist. This then diminishes the incentives of other consumers to use the entrant's product. Together, these effects then reduce the ability of the entrant to either profit on its own or receive a high acquisition price from the incumbent, diminishing the incentive to invest. Mermelstein et al. (2020) develop a computational model of mergers of substitutes that generate efficiencies from scale economies and also induce new entry. Among other interesting findings, most related to this paper is that a more restrictive merger policy can imporove welfare by limiting inefficient substitute entry. Hollenbeck (2020) develops a computational model of mergers, entry, and quality-improving investment that finds that allowing mergers that reduce current consumer surplus can sometimes raise long-term consumer surplus by inducing entry that increases investment and competition in the long-run. This occurs if entry costs are low and firms can generate rapid innovations.

Closer to this paper, Gilbert and Katz (2021) also analyzes the effect of merger policy on the type of investment a potential entrant will make. In their model, the entrant chooses whether to locate at the same location as the incumbent on a Hotelling line or at the opposite end. They find that if mergers are prohibited, an entrant with a large quality advantage will choose to locate right next to the incumbent. Nonetheless they find that without mergers, and entrant is generally biased towards too much horizontal differentiation relative to maximizing total surplus, but could be biased in either direction relative to maximizing consumer surplus. If mergers are allowed, however, then the entrant

has less incentive to differentiate horizontally if that decreases the duopoly profits more than it reduces merged profits. In their model, the antitrust authority can perfectly observe the type of entry. As with this paper, they show that maximizing ex ante welfare often requires a different merger policy than maximizing ex post welfare.

Letina et al. (2020) also model the choice of innovation projects by an entrant, but all of the choices are among potential substitutes. They use the model to show the circumstances under which allowing mergers is important for increasing innovation. Callander and Matouschek (forthcoming), by contrast, model the innovation decision of a potential entrant along a hotelling line and show that the possibility of a merger induces the entrant to choose innovations that are closer substitutes for the incumbent's product than if mergers were not allowed.

Katz (2021) shows that stricter merger policy increase an entrant's incentive to invest in marginal product improvements and also when it reduces the incumbent's incentive to invest to reduce the gain from mergers. Fumagalli et al. (2020) also model the effect of merger policy on investment. They focus on the case of a financially constrained entrant, who might not be able to develop a (substitute) project without merging. They show that conditioning merger policy on the offer made by the incumbent can sometimes allow only those mergers in which the product wouldn't be developed without it. Cunningham et al. (2020) develop a model that shows how mergers can be used by incumbents to prevent nascent competitors from undermining their market power and demonstrate empirically that incumbent firms use this strategy in the pharmaceutical industry. Hemphill and Wu (2020) discuss this issue informally and consider how antitrust law might respond. See Kaplow (forthcoming) for a discussion of how these issues should, but often do not, influence merger policy.

More broadly, this paper is also related to analysis of the burden of proof when individuals can choose between harmful and benign actions, for which Kaplow (2012) is the seminal contribution.

The next section sets up the model and some theoretical results. Section 3 discusses the simulation and the simulation results. Section 4 concludes.

2 Model

In period 0, an entrepreneur/entrant (E) chooses to invest in a new market. In so doing, she can choose whether to attempt to develop a substitute product to the dominant, incumbent firm (I), develop a complementary product, or do nothing. Let p_s and p_c be the probability that E's investment will succeed in developing the substitute and competitive product, respectively. The cost of obtaining these probabilities is given by k.

In period 1, I observes which product E is developing (if any), but the product's success is not yet determined. I then makes an offer to buy E. If allowed, I and E merge and split the surplus equally. We assume that the probability of success of E's product is independent of whether there is a merger

or not. If there were synergies such that a merger would increase E's probability of success, this would naturally reduce the optimal threshold for approving the merger.

Also in period 1, the competition authority (A), which does not observe E's type of investment, observes an imperfect signal of it, σ. We assume it is imperfect to capture the common justification for allowing dominant firms to acquire new entrants, the argument that the new entrants provide complements rather than substitutes to the dominant firm's offerings. The distribution function for the signal is F_c if E made a complement investment in period 0 and F_s if E made a substitute investment (there is no merger if E did not develop any product). The associated density functions are f_c and f_s, respectively. We assume that f_s/f_c is strictly increasing, so that higher signals indicate the merger is more likely to between substitutes. A then blocks or allows the merger based on its signal.

In period 2, profit and welfare are realized. If I and E have merged, let π_m^c (π_m^s) be the joint profit if the complementary (substitute) product was successful and π_M(for monopoly) be the joint profit if neither was or if there was no merger. The associated total welfare is W_m^c, W_m^s and W_M (at this point, these can represent total welfare or consumer welfare). If there was no merger, then π_M and W_M are M's profit and total welfare if E is unsuccessful in both products (E earns zero profit). If there was no merger and E's complement (substitute) product is successful, then profits for the two firms are $\pi_I^{d,c}$ and $\pi_E^{d,c}$ ($\pi_I^{d,s}$ and $\pi_E^{d,s}$), and total welfare is $W^{d,c}$ ($W^{d,s}$). We make the following assumptions about these values:

(a) $\pi_m^s > \pi_I^{d,s} + \pi_E^{d,s} \geq 0$
(b) $\pi_m^c > \pi_I^{d,c}; \pi_m^c > \pi_I^{d,c} + \pi_E^{d,c} > 0$
(c) $W^{d,s} > W_m^s \geq W_M$ and
(d) $W_m^c > W^{d,c} > W_M$

Because we assume an equal split of the surplus from a merger, we define $\pi_E^{m,i} = p_i[\pi_E^{d,i} + (1/2)(\pi_m^i - \pi_I^{d,i} - \pi_E^{d,i})]$ as the entrant's expected gross profit from a merger when it invests in developing a product of type i. Assume that the regulator commits to a rule that allows the merger if and only $\sigma < \overline{\sigma}$; that is, it approves the merger if the signal that the merger is between complements is strong enough. Given this, the expected gross profit from an investment of type i (not knowing if a merger will be approved or not) is $\pi_E^i = F_i(\overline{\sigma})\pi_E^{m,i} + (1 - F_i(\overline{\sigma}))p_i\pi_E^{d,i}$. That is, with probability $F_i(\overline{\sigma})$ the merger is approved, in which case E gets her merger payoff. With probability $1 - F_i(\overline{\sigma})$ the merger is rejected, then E gets her duopoly profit if the product is developed successfully.

Thus, we can determine cutoff probabilities for when E will choose to develop no project, project c or project s.

Lemma 1 *(i) E will develop project c if and only if* $\frac{p_c}{p_s} \geq \frac{2\pi_E^{d,s} + F_s(\overline{\sigma})(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})}{2\pi_E^{d,c} + F_c(\overline{\sigma})(\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c})} \equiv z*(\bar{\sigma})$ *and* $p_c \geq \frac{2k}{2\pi_E^{d,c} + F_c(\overline{\sigma})(\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c})} \equiv \overline{p}_c$*; (ii) E will develop project s if*

and only if $\frac{p_c}{p_s} < z^*(\bar{\sigma})$ *and* $p_s \geq \frac{2k}{2\pi_E^{d,s} + F_c(\overline{\sigma})(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})} \equiv \overline{p}_s$; *(iii) E will develop neither project if and only if* $p_c < \overline{p}_c$ *and* $p_s < \overline{p}_s$.

Proof. E develops product i if and only if it is more profitable than developing product $-i$ and its expected profit covers its development cost, k. The condition on the ratio of $\frac{p_c}{p_s}$ follows from the fact that E only develops the complement if it is at least as profitable than the substitute, $\pi_E^c \geq \pi_E^s$ in part (i) the reverse in part (ii). The condition in (i) on only p_c follows from the fact that $\pi_E^c \geq k$ is necessary for E to develop the complement rather than not invest, while the condition in (ii) on only p_s follows from $\pi_E^s \geq k$ to induce investment in the substitute. If neither of these conditions are satisfied, as in part (iii), then neither product is profitable to develop. ■

From Lemma 1, we can directly see some effects of stricter merger enforcement (lower $\bar{\sigma}$). Because mergers happen only if it is jointly profitable, we know that $\pi_m^i - \pi_I^{d,i} - \pi_E^{d,i} > 0$. This means that stricter enforcement increases the required probability of success for either type of investment. Assuming the substitute products are close enough substitutes, if merger enforcement is not too strict, then stricter merger enforcement will encourage the development of the complementary product relative to the substitute product because the substitute product is more likely to generate the larger signal that will cause the agency to block the merger. On the other hand, if merger enforcement is already very strict, then making merger enforcement even stricter will actually discourage the development of the complementary product relative to the substitute because a merger with the substitute product is already very unlikely to be allowed–so the effect of even stricter enforcement will mostly just affect the complementary product. The following corollary collects these effects.

Corollary 2 $\overline{p}_i$ *is decreasing in* $\overline{\sigma}$ *for* $i = \{c, s\}$ *(weaker merger enforcement reduces the probability threshold necessary for either investment to be profitable); there exists a* $\widehat{\sigma}$ *such that* $z*(\bar{\sigma})$ *is increasing in* $\overline{\sigma}$ *if and only if* $\overline{\sigma} > \widehat{\sigma}$. *If* $\frac{\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s}}{\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c}} > \frac{\pi_E^{d,s}}{\pi_E^{d,c}}$ *(the substitute product is a close enough substitute), then* $\widehat{\sigma}$ *is small enough that* $f_c(\widehat{\sigma}) < f_s(\widehat{\sigma})$. *That is, weaker merger enforcement increases the threshold probability of success for c relative to s necessary for c investment to be more profitable than s investment if and only if merger enforcement is not too strong).*

Proof. See Appendix. ■

Next, consider how E's investment decision is biased relative to expected social welfare. The expected gross social welfare from a type i investment is s $= (1-p_i) + p_i F_i(\overline{\sigma}) W_m^i + (1 - F_i(\overline{\sigma}))p_i$. With probability $1-p_i$, the investment fails and we get the welfare associated with only I producing (whether or not there is a merger). If the investment is successful, then we get the welfare associated with the merger of the two products if it is approved (probability $F_i(\bar{\sigma})$) and from duopoly if the merger is not approved (probability $(1 - F_i(\overline{\sigma}))$). To determine the extent and direction of any bias in E's investment decision

relative to the social optimum, we evaluate the difference in expected welfare for the two types of investment when they produce equal profit ($\frac{p_c}{p_s} = z * (\bar{\sigma})$):

$$(W^s - W^c \mid \frac{p_c}{p_s} = \frac{2\pi_E^{d,s} + F_s(\overline{\sigma})\Delta\pi_m^s}{2\pi_E^{d,c} + F_c(\overline{\sigma})\Delta\pi_m^c} = \frac{p_s}{2\pi_E^{d,c} + F_c(\overline{\sigma})\Delta\pi_m^c} \{(2\pi_E^{d,c} + F_c(\overline{\sigma})\Delta\pi_m^c)[(1 - F_s(\overline{\sigma}))W^{d,s} + F_s(\overline{\sigma})W_m^s - W_M] - (2\pi_E^{d,s} + F_s(\overline{\sigma}))\Delta\pi_m^s)[(1 - F_c(\overline{\sigma}))W^{d,c} + F_c(\overline{\sigma})W_m^c - W_M]\} \quad (1)$$

Where we define $\Delta\pi_m^c \equiv \pi_m^c - \pi_I^{d,c} - \pi_E^{d,c}$ and $\Delta\pi_m^s \equiv \pi_m^s - \pi_I^{d,s} - \pi_E^{d,s}$. This has the sign of the term in curly braces. The first line in curly braces is the entrant's expected profit from developing a complement times the expected welfare gain if the entrant develops a substitute. The second line in curly braces is the entrant's expected profit from developing a substitute times the expected welfare gain if the entrant develops a complement. If merger enforcement is very lax, $\overline{\sigma}$ is close to one, then the curly braces term is clearly negative because $W_m^s - W_M < 0$. Thus, under very lax merger control, there is excessive incentive to develop the substitute product. On the other hand, if merger enforcement is very strict, $\overline{\sigma}$ is close to zero, then this is positive if and only if $\frac{W^{d,s} - W_M}{\pi^{d,s}} > \frac{W^{d,c} - W_M}{\pi^{d,c}}$. That is, there is excessive incentive to develop the complement product under very strict merger enforcement if and only if the welfare gain under duopoly from developing a substitute product per unit profit exceeds the welfare gain from developing a complement product per unit profit under duopoly. This will be true if the products are close enough substitutes.

This suggests that there is a unique signal cutoff, $\overline{\sigma}^*$, such that if merger enforcement is very strong ($\overline{\sigma} < \overline{\sigma}^*$) then there will be an excessive incentive to develop the complement product relative to social welfare, but that the reverse will be true for merger enforcement that is stricter than this cutoff value ($\overline{\sigma} > \overline{\sigma}^*$). The next result proves that this is the case.

Proposition 3 *If* $\frac{W^{d,s} - W_M}{\pi^{d,s}} > \frac{W^{d,c} - W_M}{\pi^{d,c}}$ *, there is a unique* $\overline{\sigma}^*$ *such that there is an excess private incentive to choose the substitute product if and only if* $\overline{\sigma} > \overline{\sigma}^*$ *(merger enforcement is relatively weak).*

Proof. See Appendix. ■

This means that in terms of ex ante incentives, there is an interior optimum merger strictness for aligning ex ante choice between investments with social welfare. This doesn't necessarily guarantee that $\overline{\sigma}^*$ generates optimal ex ante investment incentives since we also have to consider the decision to invest or not. The next result shows that at the $\overline{\sigma}^*$ that generates socially optimal incentives to choose between the complement and substitute investment, there is insufficient incentive to invest in either product.

Proposition 4 *Let $\overline{\sigma}^*$ be the merger cutoff such that the substitute investment is more profitable than the complement investment if and only if it generates greater expected social welfare. Then at $\overline{\sigma}^*$, the expected welfare from investment in either product is strictly positive when the expected profits from investment in both products are zero.*

Proof. See Appendix. ■

This result suggests a fundamental conflict between optimizing the entrant's incentive to choose between investing in the substitute and complement product and optimizing its incentive to invest at all. The second best optimal investment incentives, then, must involve a merger standard that generates some bias towards the substitute investment along with somewhat insufficient incentives to invest at all. Of course, even this does not resolve the question of the optimal standard because social welfare depends not just on ex ante investment incentives but also on ex post efficiency. We now turn to analyzing the overall second best optimal mergers standard combining all three issues (the two ex ante investment incentives along with ex post efficiency).

Let G_i be the cumulative distribution function for p_i, with g_i the associated probability density function; $i \in \{c, s\}$. Our social welfare function is as follows:

$$G_s(\overline{p}_s)\int_{\overline{p}_c}^{1} g_c(p_c)\{p_c[F_c(\overline{\sigma})W^{m,c} + (1-F_c(\overline{\sigma}))W^{d,c} - W^m] - \delta k\}dp_c + \quad (2)$$

$$\int_{\overline{p}_s}^{1} G_c(z*(\bar{\sigma})p_s)g_s(p_s)\{p_s[F_s(\overline{\sigma})W^{m,s} + (1-F_s(\overline{\sigma}))W^{d,s} - W^m] - \delta k\}dp_s +$$

$$\int_{\overline{p}_s}^{1}\{\int_{z*(\bar{\sigma})p_s}^{1} g_c(p_c)(p_c[F_c(\overline{\sigma})W^{m,c} + (1-F_c(\overline{\sigma}))W^{d,c} - W^m] - \delta k)dp_c\}g_s(p_s)dp_s$$

The first term reflects when p_s is too low for the substitute investment to be profitable. In that case, welfare comes from when the probability of success for the complement investment is high enough to generate positive profits. When that occurs, the social welfare gain comes from when the investment is successful. In that case, if the merger signal is below the threshold, we get welfare from a complement merger, and when it is above we get welfare from a complement duopoly (both of which are compared to the monopoly welfare without the complement). We then subtract off δ times the cost of the investment, k. Here, $\delta \in \{0, 1\}$ is an indicator variable. If we are using a total welfare standard, then $\delta = 1$ because we count the investment cost as a social cost. If, instead, we are using a consumer welfare standard, then $\delta = 0$ because the investment cost does not reduce consumer welfare (and the welfare terms represent consumer welfare rather than total welfare).

The second term reflects when p_s is both large enough for the substitute investment to be profitable and large enough relative to p_c for it to be more profitable than the complement investment. (The probability that p_c is that low is given by $G_c(z*(\bar{\sigma})p_s)$.) In that case, if the substitute investment is successful, and if the merger signal is below the threshold, we get welfare from

a substitute merger. If the investment is successful and the signal is above the threshold, then we get welfare from a substitute duopoly (both of which are compared to the monopoly welfare without the substitute, and again subtracting off the investment cost under a total welfare criterion).

The last term reflects when both investments are profitable, but the complement investment is more profitable ($p_c > z*(\bar{\sigma})p_s$). In that case (again, when the investment is successful), if the merger signal is below the threshold, we get welfare from a complement merger, and when it is above we get welfare from a complement duopoly (both of which are compared to the monopoly welfare without the complement, again subtracting off the investment cost under a total welfare criterion).

We can now take the derivative of (2) with respect to $\overline{\sigma}$ to see the total effect of loosing merger regulation on social welfare:

$$
\begin{aligned}
&-\frac{d\overline{p}_s}{d\overline{\sigma}}\{\overline{p}_s[F_s(\overline{\sigma})W^{m,s}+(1-F_s(\overline{\sigma}))W^{d,s}-W^m]-\delta k\}G_c(\overline{p}_c)g_s(\overline{p}_s) \qquad (3)\\
&-G_s(\overline{p}_s)\frac{d\overline{p}_c}{d\overline{\sigma}}\{\overline{p}_c[F_c(\overline{\sigma})W^{m,c}+(1-F_c(\overline{\sigma}))W^{d,c}-W^m]-\delta k\}g_c(\overline{p}_c)+\\
&\int_{\overline{p}_s}^1 G_c(z*(\bar{\sigma})p_s)f_s(\overline{\sigma})(W^{m,s}-W^{d,s})g_s(p_s)p_s dp_s+\\
&G_s(\overline{p}_s)\int_{\overline{p}_c}^1 g_c(p_c)p_c f_c(\overline{\sigma})(W^{m,c}-W^{d,c})dp_c+\\
&\int_{\overline{p}_s}^1\{\int_{z*(\bar{\sigma})p_s}^1 g_c(p_c)p_c f_c(\overline{\sigma})(W^{m,c}-W^{d,c})dp_c\}g_s(p_s)dp_s+\\
&\int_{\overline{p}_s}^1\{p_s[F_s(\overline{\sigma})W^{m,s}+(1-F_s(\overline{\sigma}))W^{d,s}-W^m]\\
&-z*(\bar{\sigma})p_s[F_c(\overline{\sigma})W^{m,c}-(1-F_c(\overline{\sigma}))W^{d,c}-W^m]\}g_c(z*(\bar{\sigma})p_s)\frac{dz*(\bar{\sigma})}{d\overline{\sigma}}p_s g_s(p_s)dp_s
\end{aligned}
$$

The two terms on the first line reflect the expected welfare gain from fact that looser merger approval will increase the incentive to invest (in the substitute product in the first term and the complement product in the second). These have the sign of the terms in the square brackets because $\frac{d\overline{p}_i}{d\overline{\sigma}} < 0$, more lax merger approval lowers the necessary probability of success to make investment profitable. Under the consumer welfare standard, these terms are necessarily positive because consumer welfare can only increase as a result of entry. Under the total welfare standard, however, these could be negative. In particular, if the substitute is a perfect substitute, then welfare under a substitute merger is no greater than without entry. Thus, if the cost of the investment exceeds the expected welfare gain from greater competition when the merger is prohibited, greater investment incentives could reduce social welfare.

The term on the third line is the ex post effect of looser merger approval when there is a successful substitute investment. This is negative because approving a substitute merger lowers welfare ($W^{m,s} - W^{d,s} < 0$). The fourth and fifth

lines are the ex post effect of looser merger approval when there is a successful complement investment. The fourth line reflects when the substitute investment is not profitable, the fifth line when it is profitable but less profitable than the complement investment. The last two lines reflect the welfare effect of looser merger approval resulting from the entrant switching its type of investment. If we use the definition of $z*(\bar{\sigma})$ in this line, we can see that the term in curly braces is exactly the same as expression as (1). Thus, it is positive if and only if $\overline{\sigma} < \overline{\sigma}^*$ (by Lemma 4). By Corollary 3, $\frac{dz*(\bar{\sigma})}{d\bar{\sigma}} > 0$ if and only if $\overline{\sigma} > \widehat{\sigma}$.

Unfortunately, without putting more structure on the model, we cannot make much more headway into how considering the ex ante effects of merger review standards on incentives to invest should affect the optimal choice of these standards. Even with more structure, the first order condition cannot be solved analytically. So, in the next section, we proceed via numerical simulation.

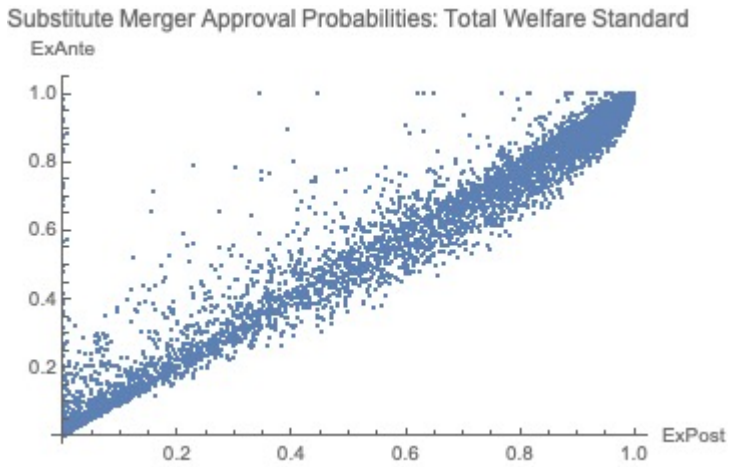
3 Simulation

In our simulation, we assume that the probability of success for each type of product is uniformly distributed between zero and one; $G_s(p) = G_c(p) = p$. We also assume the signals received by the antitrust authority have the density functions $f_s(\sigma) = 2\sigma$ and $f_c(\sigma) = 2(1-\sigma)$. There is a representative consumer whose utility function is given by $U = (q + q_s + q_c) - [(q^2 + q_s^2 + q_c^2) + 2zqq_s + 2a(q+q_s)q_c]/2 - Pq - P_s q_s - P_c q_c$. Here, q is the quantity of the incumbent's product the consumer purchases, while q_i is the quantity of the entrant's product of type $i \in \{c, s\}$ (note, that only one of these is positive since the entrant has to choose to develop one or the other of these products). P is the price of the incumbent's product, while P_i is the price of the entrant's product of type i. This creates a linear demand curve for the incumbent's product of $1-p$ without entry. With substitute entry, the demand functions are $q = \frac{1-P-z(1-P_s)}{1-z^2}$ and $q_s = \frac{1-P_s-z(1-P)}{1-z^2}$. With complement entry, the demand curves are $q = \frac{1-P-a(1-P_c)}{1-a^2} q_c = \frac{1-P_c-a(1-P)}{1-a^2}$. The marginal costs of production for the incumbents and each type of entrant are c, c_s, and c_c.

In this setting, after solving for the profit-maximizing prices with and without a merger for both types of products, one can generate explicit expressions for profits, welfare and consumer welfare. Those expressions are given in the appendix. We use these expressions in the first order condition for the optimal merger standard (3) for both a total welfare standard and a consumer welfare standard and solve this for the optimal cutoff signal, $\overline{\sigma}$. We can also solve for the ex post optimal cutoff signal by just considering the second and third lines of (3). This can't be solved analytically because it is a polynomial of degree larger than four in $\overline{\sigma}$. But, we can solve it numerically by randomly drawing values for our remaining parameters: c, c_c, c_s (the costs of the incumbent, the entrant's substitute product, and the entrant's complement product, respectively), a (the degree of complementarity for the complement product), z (the degree of substitutability for the substitue product), and k (the cost of investment).

We draw randomly in $z \in [0,1]$ (0 representing independent products and 1 perfect substitutes); $a \in [-.9, 0]$ (we limit the complementarity to $-.9$ to keep profit, demand, and welfare in a reasonable range); $c \in [0,1]$; $c_s \in [max\{0, \frac{-2(1-c)+z}{z}\}, \frac{1+c}{2}]$ (the lower bound ensures that incumbent still provides a competitive constraint–otherwise there is no social loss from the merger–and the upper bound ensures that entrant provides some competitive constraint–otherwise there is no potential profit from entry with the substitute even if a merger is allowed); $c_c \in [max\{0, \frac{-2(1-c)+z}{z}\}, 1]$ (we allow the maximum cost of the complement to exceed the substitute to reflect the fact that it should generally be easier to produce a substitute); and k between 0 and the lesser of the profit the entrant could earn with a substitute product and complement product if the merger was always allowed (so that it is always possible to choose a merger cutoff such that the entrant chooses both products with positive probability). We did 10,000 draws from these distributions and calculated the optimal merger cutoff for both a total welfare standard and a consumer welfare standard, for both ex ante and ex post welfare maximization.

The following figure plots the probability of approving a substitute merger under the optimal ex ante and ex post cutoff under the total welfare standard for each of the 8701 draws in which $z \geq 1/8$. We exclude the cases with $z < 1/8$ because the maximum percentage price increase from the substitute merger at $z = 1/8$ is about 7% if marginal costs are zero for both the incumbent and entrant. In general, for $z < 1/8$, the cost increase is well below the $5 - 10\%$ threshold under the Merger Guidelines.



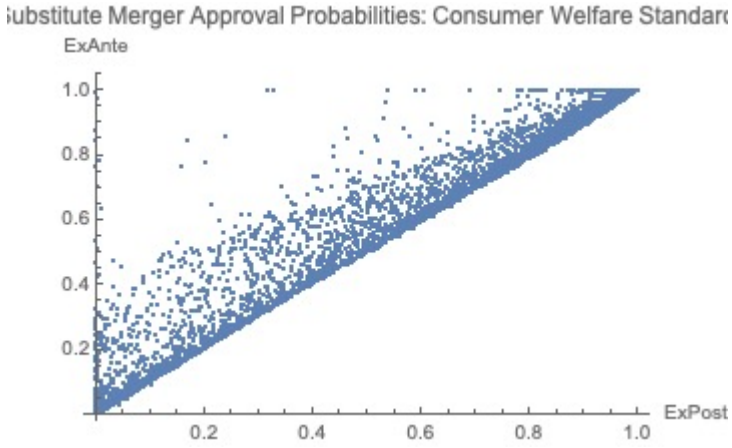
Notice that, for the most part, the dots are clustered around the 45 degree line, indicating that considering ex ante investment very often does not result in a huge change in the optimal merger standard. Also, there are slightly more dots below the 45 degree line than above it, which indicates that the optimal ex

ante standard is often stricter than the optimal ex post standard. On the other hand, almost all of the dots that are quite far from the 45 degree line are above it. This means that when ex ante investment considerations do suggest large shifts in the merger standard, those large shifts are generally in the direction of more lax standards.

There are two primary reasons for why ex ante investment considerations do not necessarily suggest more lax standards are optimal. First, under the total welfare standard, there can be a socially excessive incentive to invest in a very close substitute product if merger review is not sufficiently strict because a substitute merger enables the entrant to share in some of the incumbent's profit without generating much additional ex post welfare, making the investment cost mostly a social waste. On the other hand, if the entrant develops a close substitute and is not able to merge, the entrant's profit is small, and it is mostly generating a positive externality on consumers. This makes it ex ante optimal to not have too strict a merger standard. But, the existence of the complement product already ensures this, so the dominant effect is the concern about excessive substitute product development.

Second, not only does the merger standard affect whether the entrant develops a product, it also affects what type of product it chooses to develop. If merger standards are too lax, then there is a socially excessive incentive to choose the substitute product, which may not add much to social welfare if the merger is approved (and often won't add as much as a complement under a total welfare standard even if the merger is not approved). In contrast, the complement product offers a positive externality both to the incumbent and to consumers. So, if this product is sufficiently valuable, it is important to incentivize the entrant to choose this product rather than the substitute. A stricter merger standard does this.

As the next graph shows, the effect of ex ante investment considerations under the consumer welfare standard is much clearer.



Comparison case	c	c_s	$c-c_s$	c_c	z	a	k	$\overline{\sigma}^2_{EA}$	$\overline{\sigma}^2_{EP}$
Overall sample means	0.50	0.44	0.06	0.56	0.50	-0.45	0.03	0.45^{tw}, 0.42^{cw}	0.44^{tw}, 0.39^{cw}
TW: subst approval 10% more	ND	0.39	0.13	0.71	0.54	ND	0.05	0.46	0.18
TW: substi approval 10% less	0.37	0.40	-0.03	0.62	0.83	-0.53	ND	0.63	0.75
CW: subst approval 10% more	0.43	0.42	0.01	0.63	0.71	$-.33$	0.04	0.53	0.28

While we still see a larger cluster around the 45 degree line, most of the deviations, and all of the significant deviations, are above the line. Thus, under a consumer welfare standard, considerations of ex ante investment have a much stronger tendency to warrant more lax merger standards than under the total welfare standard.

It is somewhat surprising that taking into account the effect of merger standards on ex ante investment incentives is more likely to lead to less strict merger review under a consumer welfare standard than under a total welfare standard. There are a number of potential explanations for this. First, less strict merger standards encourage investment of both types. Because the investment cost is a social cost only under the total welfare standard, this tends to make the investment more valuable under the consumer welfare standard. Furthermore, if this additional investment is in the substitute product, it will likely contribute more to consumer welfare than to total welfare in that it will often cost the incumbent as much or more than it will benefit the entrant, so total profit will fall if the products are close enough substitutes. Of course, if the additional investment induced is a complement investment, the reverse is true. But, given that the optimal merger standards in the simulations are mostly $\bar{\sigma} > 1/2$, relaxing these standards is more likely to benefit a substitute investment than a complement one. Second, looser merger standards cause the entrant to switch from a complement to a substitute investment more frequently than the reverse, and substitute investments contribute relatively more to consumer welfare than total welfare compared to complement investments.

The next table shows how the parameters differ in the cases in which the ex ante and ex post optimal standards are substantially different (a greater than 10 percentage point difference in the probability of approving a substitute merger) relative to the overall sample. ND indicates no significant difference. All subsample means that are provided differ significantly from overall sample means at a p-value of less than 0.01, except for c_s when approval is at least 10 percentage points more likely ex ante under the consumer welfare standard, which is only significantly different from the overall sample at a p-value of 0.017.

One of the clearest implications of these results is that substantially more lenient ex ante merger standards (relative to the ex post optimum) are associated with substantially more costly investment and greater complement costs (suggesting that when complement entry is less profitable is is desirable to relax

merger standards to incentivize complement entry). This holds under either welfare standard. Under the total welfare standard, substantially stricter ex ante standards are associated with lower costs for both the entrant and the substitute product (duopoly is more profitable). Under a consumer welfare standard, substantially more lenient ex ante standards is associated with much closer substitutability. This suggests that when the substitute products are very close substitutes, it may be very difficult to induce entry unless there is a decent probability of merger. The entry of a close substitute generates a larger increase in consumer welfare than total welfare, thus it is more valuable to risk approving a welfare reducing merger to induce entry under the consumer welfare standard.

The last two columns of the table show how the average probability of approval of a substittute merger differs. In the overall sample, as the scatter plots above suggest, the average probability of approval is quite similar under the ex ante and ex post optimal standards. In those cases in which the ex ante optimal threshold is substantially more lenient than the ex post optimal threshold under a total welfare standard, this is almost entirely due to cases in which the ex post optimal standard is much stricter than average. This is also the case, although to a lesser degree, under the consumer welfare standard. By contrast, in those cases in which a total welfare standard suggests a much stricter ex ante threshold than ex post threshold, the threshold for both is much more lenient than average.

To further explore the effects of the parameters on the how ex ante considerations affect the optimal cutoff, we also examine subspaces of the parameter space informed by the above table. Under the total welfare standard, 144 of the 398 cases in which the ex ante optimal standard has a probability of approval of a substitute merger that is at least 10 percentage points higher occur in the 968 draws in which both the investment cost and the complement marginal cost are both above their means (because draws require that both investments are profitable, there are disproportionately fewere draws in which both k and c_c are high). Furthermore, the average difference in the probability of a substitute merger between the ex ante and ex post optimal standards in these 968 draws is about 8 percentage points. If we further restrict the region so that $k > 0.37$ and $c_c > 0.66$, then the average difference in the probability of a substitute merger between the ex ante and ex post optimal standards rises to over 15 percentage points.

We can further support the claim that, under the total welfare standard, ex ante investmet considerations warrant more lenient merger standards when the investment cost is high and the profitiability of the complement investment is low by examining the subspace of our sample in which the entrant's complement duopoly profit is less than 3/2 the entry cost (so that entry for duopoly would only be profitable if the probability of success were at least 2/3). There are just under 1500 cases in this region, but this region contains 305 of the 398 cases in which the the ex ante optimal standard has a probability of approval of a substitute merger that is at least 10 percentage points higher than the ex post optimal standard. The difference in the average probability of approval of a substitute merger in this region is 12.4% versus 6.5%. If we further restrict the

region by also requiring a welfare gain from a complement merger of at least 0.01 (the overall mean gain is 0.086), we still capture 222 of the cases with a 10 percentage point greater probability of substitute merger approval and the mean difference in the probability of substitute merger approval is 26.5% compared to 14.5%.

Unfortuantely, it is much harder to isolate the regions of parameter space for which, unter the total welfare standard, the optimal ex ante cutoff is substantially stricter than the optimal ex post cutoff. If we examine a subspace in which $\{c < 0.5, c - c_s < 0, x > 0.5\}$, the average probability of approval of a substitiute merger is about 4.5 percentage points lower under the ex ante optimal standard than under the ex post optimal one (87.2% v. 91.6%). There are 631 total cases in this region, and it contains 55 of the 285 total cases in which the probability of approval of a substitute merger is at least 10 percentage points lower under the ex ante optimal standard.

Under the consumer welfare standard, a large driver for substantially more lenient ex ante standards seems to be the need to induce substitute entry. If we restrict our analysis to the roughly 35% of the cases in which the entrant's substittue duopoly profit is less than 50% greater than its entry cost (so substittue entry for duopoly is only profitable if the probability of success is at least 2/3), the average difference in the probability of approval of a substitute merger under the consumer welfare optimal ex ante standard is about 7.5 percentage points greater than under the ex post standard. Similarly, this region contains 643 of the 750 total cases in which the ex ante consumer welfare optimal standard has a 10 percentage point greater probability of approval of a substitute merger than the ex post consumer welfare optimal standard.

Taken together, the results of simulations suggest the following.

Remark 5 *(a) There are a large range of parameter values for which ex ante investment effects do not significantly affect the optimal merger standard.*

(b) Ex ante investment effects warrant substantially more lenient merger standards much more frequently under a consumer welfare standard than under a total welfare standard. Under a total weflare standard, ex ante effects can warrant both substantially stricter and more lenient merger standards depending on parameter values.

(c) Ex ante investment effects often justify more lenient merger standards under a total welfare standard when they are necessary to induce complement entry.

(d) Ex ante investment effects often justify more lenient merger standards under a consumer welfare standard when they are necessary to induce substitute entry.

4 Conclusion

This paper models the welfare effects of merger review when a dominant firm acquires a potential competitor. The model explicitly acknowledges that the

competition authority only has an imperfect signal of whether this merger represents a merger of a potential competitor or a potential complement. The former being ex post welfare reducing while the latter is ex post welfare increasing. The model also considers how this merger review affects the incentive of the potential entrant to invest in developing a new product and, if it decides to invest, whether to choose to develop a substitute or complement product. Thus, the prior probabilities of each type of entry and merger are endogenously determined by the merger review standard.

Using this model, the paper provides some general results about how merger review standards affect investment incentives. Stricter standards discourage all types of investment, but (unless the standard is very strict) they discourage substitute investment more than complement investment. There is a review standard that provides socially optimal incentives to choose the right type of investment, but at this standard, there are suboptimal incentives to invest at all. The paper then uses a numerical simulation of the model to determine how the socially optimal merger review standard, taking into account both investment incentives and ex post welfare, compares to the standard that only optimizes ex post welfare. These results reinforce the intuition from the model. Optimal ex ante merger standards can be either more or less lenient than the ex post optimum, although under a consumer welfare standard there is a greater tendency for the ex ante optimal standard to be more lenient than the ex post standard than is the case under the total welfare standard. In fact, in our simulations, we did not find any parameters for which the optimal ex ante standard was substantially more strict (greater than 10 percentage points decrease the probability of approval of a substitute merger), while this did occur in over 2.5% of the cases under a total welfare standard. This is largely because both inducing investment is more valuable under a consumer welfare standard and there is less reason to want to encourage complement investment over substitute investment under a consumer welfare standard. Overall, our simulations show that taking into account ex ante investment incentives sometimes substantially change the optimal merger standard, but in many cases do not.

5 Appendix

Proof. Corollary 2. $\pi_m^i - \pi_I^{d,i} - \pi_E^{d,i} > 0$ implies that $\frac{2k}{2\pi_E^{d,i} + F_i(\overline{\sigma})(\pi_m^i - \pi_I^{d,i} - \pi_E^{d,i})}$ is decreasing in $\overline{\sigma}$. The derivative of $z*(\bar{\sigma}) = \frac{2\pi_E^{d,s} + F_s(\overline{\sigma})(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})}{2\pi_E^{d,c} + F_c(\overline{\sigma})(\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c})}$ with respect to $\overline{\sigma}$ is:

$$\frac{\begin{array}{c}(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})(\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c})[F_c(\overline{\sigma})f_s(\overline{\sigma}) - F_s(\overline{\sigma})f_c(\overline{\sigma})] + \\ 2\{(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})\pi_E^{d,c} f_s(\overline{\sigma}) - (\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c})\pi_E^{d,s} f_c(\overline{\sigma})\}\end{array}}{[2\pi_E^{d,c} + F_c(\overline{\sigma})(\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c})]^2} \tag{4}$$

The denominator is positive. The first term in the numerator has the sign of the term in square brackets. $F_c(\overline{\sigma}) \geq F_s(\overline{\sigma})$ always and $f_s(\overline{\sigma}) > f_c(\overline{\sigma})$ if and only

if $\overline{\sigma}$ is large enough. Thus, the first term is positive if and only if large enough $\overline{\sigma}$ is large enough, and is always positive if $f_s(\overline{\sigma}) = f_c(\overline{\sigma})$. The term in curly braces is also positive if and only if $\overline{\sigma}$ is large enough. If $\frac{\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s}}{\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c}} > \frac{\pi_E^{d,s}}{\pi_E^{d,c}}$, this term is positive whenever $f_s(\overline{\sigma}) = f_c(\overline{\sigma})$. ■

Proof. Proposition 1. Taking the derivative of the curly braces term in (1) with respect to $F_c(\sigma)$ gives:

$$[(1 - F_s(\overline{\sigma}))W^{d,s} + F_s(\overline{\sigma})W^{m,s} - W^m)]\Delta\pi_m^c - (W^{m,c} - W^{d,c})(2\pi_E^{d,s} + F_s(\overline{\sigma}))\Delta\pi_m^s) -$$
$$\{[(1 - F_c(\overline{\sigma}))W^{d,c} + F_c(\overline{\sigma})W^{m,c} - W^m)]\Delta\pi_m^s - (W^{m,s} - W^{d,s})(2\pi_E^{d,c} + F_c(\overline{\sigma}))\Delta\pi_m^c)\}f_s(\overline{\sigma})/f_c(\overline{\sigma}) \tag{5}$$

While this is not always negative, (for $\overline{\sigma}$ near zero, one can see that it is positive), we can show that its derivative with respect to $\overline{\sigma}$ is negative. This means that since it first crosses zero from above under the condition that $\frac{W^{d,s} - W_M}{\pi^{d,s}} > \frac{W^{d,c} - W_M}{\pi^{d,c}}$, it can never become positive again and cross zero from below. This derivative is:

$$-\{[(1 - F_c(\overline{\sigma}))W^{d,c} + F_c(\overline{\sigma})W^{m,c} - W^m]\Delta_m^s + (W^{d,s} - W^{m,s})(2\pi_E^{d,c} + F_c(\overline{\sigma}))\Delta\pi_m^c)\}$$
$$*[f_c(\overline{\sigma})f_s'(\bar{\sigma}) - f_c'(\overline{\sigma})f_s(\bar{\sigma})] \tag{6}$$
$$-2[(W^{d,s} - W^{m,s})\Delta\pi_m^c + (W^{m,c} - W^{d,c})\Delta\pi_m^s]f_c(\overline{\sigma})^2 f_s(\bar{\sigma}) < 0$$

This is negative because $f_c(\overline{\sigma})f_s'(\bar{\sigma}) - f_c'(\overline{\sigma})f_s(\bar{\sigma}) > 0$ by MLRP. ■

Proof. Proposition 2. First, we find the merger cutoff such that the net expected welfare gain from the substitute product is zero exactly when the expected profit from investing in the substitute is zero. The net expected welfare when profit is zero is:

$$(W^s - W^m - k \mid p_s = \frac{2k}{2\pi_E^{d,s} + F_s(\overline{\sigma})(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})}) = \tag{7}$$
$$\frac{k[(2W^{d,s} - W^m - \pi_E^{d,s}) - (2W^{d,s} - 2W^{m,s} + \pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})F_s(\overline{\sigma})]}{2\pi_E^{d,s} + F_s(\overline{\sigma})(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})}$$

This is zero if and only if $F_s(\overline{\sigma}) = \frac{2W^{d,s} - W^m - \pi_E^{d,s}}{2W^{d,s} - 2W^{m,s} + \pi_m^s - \pi_I^{d,s} - \pi_E^{d,s})}$. Define $\overline{\sigma}^{\#}$ as the level $\overline{\sigma}$ that satisfies this condition.

Now, we substitute this into the curly braces term in (1) to find the expression that gives the sign of the welfare difference between the substitute and complement investment when both generate equal profits. This gives:

$$\frac{[(W^{d,s} - W^m)(\pi_m^s - \pi_I^{d,s} - \pi_E^{d,s}) + 2(W^{d,s} - W^{m,s})\pi_E^{d,s}]\times}{\{-2(W^{d,c} - W^m + \pi_E^{d,c}) - (2(W^{m,c} - W^{d,c}) - (\pi_m^c - \pi_I^{d,c} - \pi_E^{d,c}))F_c(\overline{\sigma})\}}$$
$$2W^{d,s} - 2W^{m,s} + \pi_m^s - \pi_I^{d,s} - \pi_E^{d,s} \tag{8}$$

The denominator and the term in square brackets are positive. The first term in the curly braces is negative because the complement investment has a positive externality. The second term in curly braces is negative because the complement merger has a positive externality. Thus, this entire expression is negative. This means that when the substitute and complement product both generate zero expected profit for the entrant and the substitute product generates zero expected welfare, the complement product must generate positive expected welfare.

Thus, at $\overline{\sigma}^*$ and p_c and p_s such that both investments generate zero expected profit, we know that the expected welfare of both are equal and can't both be zero. If both generate negative expected welfare, then increasing $\overline{\sigma}$ (relaxing merger approval standards) will increase expected welfare for the complement investment and decrease it for the substitute and also make both more profitable. So, to get back to zero profits for both, we must decrease both p_c and p_s, but this further decreases the welfare of the substitute product. Thus, if welfare is negative for both at $\overline{\sigma}$ then we know that it isn't possible for $\bar{\sigma} < \overline{\sigma}^{\#\cdot}$ Say, instead that we decrease $\overline{\sigma}$, then we must increase both p_c and p_s to get back to zero profits. Then, by Lemma 4, we know that the substitute investment generates greater welfare, which contradicts what we just showed is true at $\overline{\sigma}^{\#}$. So, it is impossible for welfare to be negative for both at $\overline{\sigma}$, so it must be positive for both. ■

Profit, welfare, and consumer welfare expressions.

After solving for the profit-maximizing prices with and without a merger for both types of products, we have the following values for profits, welfare and consumer welfare (CW):[1]

[1] In these expressions, the superscript zero denotes the case in which either the incumbent's or the entrant's product is not produced in equilibrium. This case does not arise if the entrant's product is a complement.

$$\pi_I^{d,s} = \frac{[(2-z^2)(1-c)-z(1-c_s)]2}{(4-z^2)^2(1-z^2)}; \pi_E^{d,s} = \frac{[(2-z^2)(1-c_s)-z(1-c)]2}{(4-z^2)^2(1-z^2)};$$

$$\pi_m^s = \frac{[2(1-z)(1-c-c_s+cc_s)+(c-c_s)^2]}{4(1-z)^2}; \pi_m^{s,0} = Max\{\frac{(1-c_s)^2}{4}, \frac{(1-c)^2}{4}\};$$

$$\pi_I^{d,s,0} = \frac{(1-c_s)(z(1-c)+c_s-1)}{z^2}; \pi_E^{d,s,0} = \frac{(1-c)(z(1-c_s)+c-1)}{z^2};$$

$$\pi_m^c = \frac{[2(1-a)(1-c-c_c+cc_c)+(c-c_c)^2]}{4(1-a)^2};$$

$$\pi_I^{d,c} = \frac{[(2-a^2)(1-c)-a(1-c_c)]2}{(4-a^2)^2(1-a)^2}; \pi_E^{d,c} = \frac{[(2-a^2)(1-c_c)-a(1-c)]2}{(4-a^2)^2(1-a)^2};$$

$$W^{m,s} = \frac{3[2(1-z)(1-c-c_s+cc_s)+(c-c_s)^2]}{8(1-z)^2};$$

$$W^{d,s} = \frac{2(1-c)(1-c_s)(2+z)^2(3-2z)(1-z)+(c-c_s)^2(12-9z^2+2z^4))}{2(4-z^2)^2(1-z^2)};$$

$$W^m = 3(1-c)^2/8; W^{m,s,0} = Max\{3(1-c)^2/8, 3(1-c_s)^2/8\};$$

(9)

$$W^{d,s,0} = Max\{\frac{(1-c)(2z(1-c_s)+c-1)}{2z^2}, \frac{(1-c_s)(2z(1-c)+c_s-1)}{2z^2}\};$$

$$CW^m = (1-c)^2/8; CW^{m,s} = \frac{[2(1-z)(1-c-c_s+cc_s)+(c-c_s)^2]}{8(1-z)^2};$$

$$CW^{d,s} = \frac{2(1-z)(2+z)^2(c+c_s-1)+(4-3z^2)(c^{2+}c_s^2)-2cc_sz^3}{2(4-z^2)^2(1-z^2)};$$

$$CW^{m,s,0} = Max\{(1-c)^2/8, (1-c_s)^2/8\};$$

$$CW^{d,s,0} = Max\{\frac{(1-c)^2}{2z^2}, \frac{(1-c_s)^2}{2z^2}\};$$

$$W^{m,c} = \frac{3[2(1-a)(1-c-c_s+cc_s)+(c-c_s)^2]}{8(1-a)^2};$$

$$W^{d,c} = \frac{2(1-c)(1-c_s)(2+a)^2(3-2a)(1-a)+(c-c_s)^2(12-9a^2+2a^4))}{2(4-a^2)^2(1-a^2)};$$

$$CW^{m,c} = \frac{[2(1-a)(1-c-c_s+cc_s)+(c-c_s)^2]}{8(1-a)^2};$$

$$CW^{d,c} = \frac{2(1-c)(1-c_s)(2+a)^2(1-a)+(c-c_s)^2(4-3a^2)}{2(4-a^2)^2(1-a^2)};$$

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