Strategic Delay in Patent Enforcement

Erik Hovenkamp* Jorge Lemus† John L. Turner‡

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[PRELIMINARY VERSION]

Abstract

We analyze technology adoption and investment decisions by a developer who is at risk of patent infringement. The technology may not be patented, in which case the developer is free to use it. But if the technology is patented, the patentee can strategically decide when to approach the developer and claim infringement. Incentives to delay patent enforcement are stronger for non-practicing entities, and are strongest when patentees are slightly over-rewarded. These predictions are consistent with the timing of patent litigation by non-practicing and by practicing entities. In particular, the elimination of the doctrine of laches following a Supreme Court decision in 2017 led to more delay by non-practicing entities but not by practicing entities.

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*University of Southern California. Email: eriknhovenkamp@gmail.com

†University of Illinois at Urbana-Champaign. Email: jalemus@illinois.edu.

‡University of Georgia. Email jlturner@uga.edu

1 Introduction

The patent system is designed to encourage innovation by rewarding inventors of technology with exclusivity. Inventors may realize rewards either through earning high profits from market activity or from other forms of monetization, such as licensing revenue or court-imposed damages for infringement. In recent years, economists and policymakers have expressed concern about firms that focus on the latter approach—they are often called non-practicing entities, patent assertion entities, or patent trolls.[1] Such firms have been accused of seeking overly generous rewards by being "trolls" and by pursuing "holdup" strategies, and theory predicts over-rewarded patentees in certain situations (Lemley and Shapiro, 2006).

Patent monetization, however, is not automatic. A patentee needs to identify firms that are interested in buying a license before choosing the technology in their products, or firms that are already selling a product that infringes their patents.[2] The 2016 FTC study on Patent Assertion Entity activity reports: "PAE activity therefore results in what often are referred to as ex post patent transactions because any patent license or settlement occurs after someone has developed or marketed the product at issue. This contrasts with ex ante patent transactions in which the technology and related patent rights transfer from an inventor to a manufacturer before the product is developed and marketed."

Our key insight is that whether patent transactions occur ex post or ex ante is *endogenously* determined in equilibrium. First, the patentee identifies a potential technology user. Subsequently, the patentee strategically decides to either approach immediately and negotiate an ex ante license or to wait and claim patent infringement after the technology choice has been made, which results in ex post licensing under the threat of litigation. This, in turn, affects innovation investments and adoption decisions by potential users.

We introduce and analyze a game of innovation, licensing, and litigation. In our baseline model, a downstream firm (the "developer") chooses one of two possible technologies to embed in a product. One of them is safe (i.e., public-domain technology). The other one delivers a higher return but it is risky: there is some ex ante chance of overlap with a patent.

[1] Non-practicing entities have existed for a long time. For example, Jerome Lemelson amassed 500+ patents and managed to secure $500 million in licensing revenue, much of this prior to 1990. NPE lawsuits were relatively uncommon before the year 2000.

[2] In practice, patentees search for such firms using online databases, going to trade shows, reading industry reports, or networking with industry experts. For recommendations on how to find potential licensee see, for example:
https://sagaciousresearch.com/blog/3-steps-detecting-patent-infringement-easy-approach/
https://www.lawfirms.com/resources/intellectual-property/licensing/soliciting-potential-licenses.htm
https://www.ipwatchdog.com/2015/12/15/systematic-approach-to-a-successful-patent-licensing-program

With some probability, however, there is no patentee and the downstream firm earns its profits unencumbered. If there is a patentee, it is informed about the downstream firm prior to the investment choice. It can either approach the firm to negotiate a license prior to investment choice or can wait to see if the downstream firm makes the risky investment and then sue. In this dynamic game of incomplete information, we use the perfect Bayesian equilibrium concept. Equilibrium beliefs about whether there is a patentee but did not approach play a significant role in our analysis.

The possibility of strategic delay in patent enforcement weakens investment incentives for developers only if (at the time of investment) there is uncertainty about whether there is a patentee and patentees would expect to be *excessively* over-rewarded whenever they get the chance to sue a developer. First, if the developer knows the patentee, then in any situation where the patentee would be reasonably rewarded, the developer gains from the risky investment even absent a license; in any situation where the patentee would be over-rewarded, the developer would credibly threaten not to undertake the risky investment, and an efficient bargain would ensue which would yield both the risky investment and a license which reasonably rewards the patentee.

Second, if the developer does not know whether there is a patentee and is not approached for a license, it will discount potential losses from the suit according to the probability that there is in fact a patentee, and will be less inclined to choose the risky investment. As a consequence, patentees that choose not to approach early may expect to be over-rewarded (i.e., earn more than the developer's extra return from taking the risk), yet the developer (who does not know there is a patentee) may take invest in the risky technology anyway. It is only when the patentee expects an excessive over-reward—more than the developer's extra return, divided by the ex ante probability that there is a patentee—that the developer will avoid the risky investment. In that case, there is a cat-and-mouse game between the patentee and the developer, and the unique mixed-strategy equilibrium entails probabilistic ex ante licensing and probabilistic inefficient investment.

The source of inefficiency in our model is novel in the literature, and a comparison with Lemley and Shapiro (2006) highlights our key contributions. They model litigation and bargaining between a patentee and downstream firm, but do not consider an investment choice with an uncertain chance of a patent. The key endogenous variable is the negotiated royalty rate, and they show this is often higher than a benchmark that would be negotiated absent any holdup. Indeed, they essentially interpret the ability of patentees to earn royalties higher than the benchmark as the main aspect of "patent holdup,"[3] and they note that this phenomenon is especially pronounced for relatively weak patents. Our model also generates, in equilibrium, average awards for patentees that are above the incremental contribution of the patented technology, and for relatively weak patents just like Lemley and Shapiro (2007) find. But because of the uncertainty around whether there is a patent, the developer's choice (for these cases) to make the risky investment is Pareto optimal and the *average* developer profits from the risky investment. Hence, over-rewarding the patentee does not necessarily yield inefficient investment.

Now, equilibrium investments may be inefficient in our model, but only when patents are relatively strong and patentees expect excessive over-rewards conditional on risky investment. And strikingly, patentees' *average* equilibrium rewards in such cases are reasonable—in fact, they are the same as the licensing fee in a bargain where the downstream firm's best outside option is the safe investment. Intuitively, the mixed strategy equilibrium specifies that a downstream firm not approached for a license chooses the risky investment probabilistically. In doing this, the patentee then earns the same from approaching for a license prior to investment as from not approaching and hoping the downstream firm chooses the risky investment. Because safe investments are Pareto inferior, this yields a welfare loss.

Our baseline model captures a number stylized facts. Equilibrium bargains that yield positive bargaining surplus occur only if the patentee expects to be excessively over-rewarded. These cases strike us as empirically uncommon.[4] Hence, we interpret this as predicting rare up-front licensing and frequent litigation by NPEs, with significant costs for developers. The surge in NPE lawsuits since the late 1990s is well known, and costs for firms sued are significant (Bessen et al 2018). Moreover, NPEs negotiate very few licenses prior to litigation against manufacturers, and those negotiations that do occur take a long time (FTC 2016).[5]

Because excessive over-rewards occur for stronger patents, our model also predicts that litigated patents will tend to be weak and litigation will follow investments. Empirically, NPE patents are weaker and asserted later than patents asserted against direct competitors (Love 2012; Cohen et al. 2019).

The model also predicts that except in rare cases, developers will essentially ignore the

[3]See also Shapiro and Lemley (2020), who discuss empirical evidence for patent holdup and antitrust considerations.

[4]The *NTP v. Research in Motion* case is also an example of excessive expected royalties.

[5]Restricting attention to patent assertion entiities (PAEs), the FTC report distinguishes between "Portfolio" PAEs (e.g., Intellectual Ventures) that tend to litigate less often and do try to negotiate licenses up front, and "Litigation" PAEs that tend to litigate more often and almost never try to negotiate licenses up front.

prospect of patent litigation in making investments. There is considerable evidence that manufacturing firms *do* generally ignore unseen risks of patent infringement when making investment decisions. Specifically, technology companies generally instruct engineers not to read patents, adopt policies of ignoring initial demand letters, and choose not to substitute non-infringing redesigns (that are ready to go) until litigation is lost (Lemley 2008).

We also consider the case where the patentee is directly harmed by the risky investment by more than the direct gain to the downstream firm. This more closely resembles the "practicing patentee" case. For this specification, it turns out that meaningful bargains do occur, but patent licensing never does. Some bargains are essentially anti-competitive in the sense that the bargain leads to the safe investment being chosen (i.e., no competition). There is also a case (moderately strong patent, relatively low chance of a patentee) where the patentee need only identify itself to get the downstream firm to switch to the safe investment. Litigation occurs in some other cases, when patents are very strong and when the likelihood of a patentee is not high. When litigation does occur, the patentee is troll-like in that it remains in the shadows and gets over-rewarded by the court.

The assumption that the patentee knows about the downstream firm's plans is strong. But classic anecdotes motivating patent holdup, i.e, getting technology approved as part of a standard prior to revealing patent ownership, involve precisely the "no approach" behavior we capture here.[6] In addition, many of our results are unchanged if nature determines the patentee's existence after the downstream firm invests. If the per-patentee reward conditional on the risky investment is not excessive, then the downstream firm will make the risky investment and payoffs are the same. The only thing that changes is that if the per-patentee reward conditional on the risky investment is excessive, then the downstream firm always chooses the safe investment. Hence, endowing the patentee with the strategic choice of whether to approach can only improve welfare.

We extend our baseline model in a number of directions. Patents in our baseline model are solely a source of risk for the developer who has all the know-how to invent B at no cost. In practice, patents provide know how and there is a cost to invent new technology. We present an extension where absent an ex-ante agreement between the developer and the patentee, the developer must pay a fixed cost to adopt technology B with some probability. The patent provide value from two sources: (1) it removes the fixed cost to access the technology and (2) it prevents failure in the adoption of technology B. Efficiency means that the patentee should always approach the developer ex ante. In equilibrium, however, the patentee does

[6] The case of Rambus has been written about extensively.

not always approach because the developer may have a strong bargaining position when negotiating ex ante (the threat of choosing technology A instead of B). Waiting removes this bargaining threat but it can harm the patentee when the developer fails to adopt B.

We also examine the possibility of costly "freedom of operate search" by the developer. We show that the developer searches only for intermediate values of royalty rates. We also study the possibility of a patent holder who arrives late. For instance, patent assertion entities may acquire old patents so it is impossible to negotiate with some developers ex ante. We show how the equilibrium technology adoption is affected by the developer anticipating the arrival of a patent assertion entity ex post.

To empirically assess the extent of strategic delay, we collect patent infringement lawsuits between 2014 and 2020 from the Stanford NPE Litigation Database and we complement this information with data from Lex Machina. Until March 21, 2017, defendants accused of patent infringement could use the "laches" defense and claim that the patentee took too long to file an infringement lawsuit. Moreover, the delay was presumed to be unreasonable if the patentee had waited more than six years after having knowledge of the alleged infringement. On March 21, 2017, however, the Supreme Court issued its decision in *SCA Hygiene Products Aktiebolag v. First Quality Baby Products, LLC*, which established that "laches" is no longer a valid defense for infringement. This ruling means that patentees have weaker incentives to file lawsuits quickly since delayed enforcement cannot be counteracted with a laches defense.

We empirically investigate the impact of removing the laches defense on delayed enforcement. Presumably, the removal of the laches defense pushes all patent asserters to use "older" patents but it may have disproportionately benefited non-producing entities because they typically assert older patents (see Figure 8). Our results show a differential impact of removing the laches defense for producing and non-producing entities. While producing entities systematically use younger patents compared to non-practicing entities, post-laches non-producing entities used even older patents. We also find that, relative to practicing entities, post-laches non-practicing entities used patents that took longer to prosecute.

These evidence points to behavior consistent with non-practicing entities having greater incentives to engage in strategic delay compared to practicing entities.

2 Model

There is a patentee (non-producing firm) and a developer (downstream firm). The developer makes an irreversible choice of incorporating one of two technologies, A and B, into a product. Both technologies cost the same, and this cost is normalized to zero. Technology A is in the public domain, so there is no infringement risk, and the developer earns a profit of $(1-\bar{\rho})\pi$ from using it, where $\bar{\rho} \in (0,1)$. Technology B, on the other hand, provides the developer a profit of π but carries some risk. With ex ante probability $\lambda \in (0,1]$, there exists a patent that reads on technology B; with probability $1-\lambda$, the developer will not face any infringement accusation.

The timing of the model is as follows. First, the patentee conducts an investigation to identify a suitable developer who may use the patented technology. The patentee privately learns the result of this investigation: with probability λ the patentee finds the developer ('informed' patentee), and with probability $1-\lambda$ the patentee does not find any suitable developer ('uninformed' patentee). An uninformed patentee does not take any action and receives 0. An informed patentee chooses whether to approach the developer and negotiate a license ("ex ante patent transaction"), or to wait for the developer to choose a technology and then demand compensation for patent infringement.[7] Waiting is valuable for the patentee because it preserves its informational advantage—the patentee is privately informed about the developer's identity.[8]

If prior to the technology choice the patentee approaches the developer, then the parties negotiate a license according to Nash bargaining under complete information; the patentee's bargaining ability is $\beta \in [0,1]$. The developer's and patentee's payoffs from this ex ante licensing negotiation are $\pi_D(\rho)$ and $\pi_P(\rho)$, respectively, which we derive in Lemma 1.

Alternatively, the patentee can wait for the developer to choose one of the technologies, and approach it only after the technology has been chosen. If the patentee waits and the developer chooses technology A, the game ends, the developer gets $(1-\bar{\rho})\pi$, and the patentee gets 0. If the patentee waits and the developer chooses technology B, then the patentee will approach the developer and request compensation for patent infringement under the threat of litigation (ex-post licensing). If litigation ensues, the patent is found valid and infringed with probability θ, in which case the patentee receives $r\pi$, where $r \in [0,1]$ is the royalty rate

[7] In Section 4 we incorporate the possibility that the patentee's investigation concludes too late for the patentee to approach the developer for a license prior to project choice. In that case, the patentee sues the developer for damages if the developer picks project B.

[8] In Section 3 we incorporate search by the developer.

set by the court. It will be convenient to work with the expected royalty rate $\rho \equiv \theta r$ that the patentee expects to receive (and the downstream firm expects to pay) when a suit is filed.

Our litigation model is simple by design. Litigation is costless.[9] We do not include legal instruments such as injunctions or triple damages, or litigation-responsive actions such as product redesigns. As previous authors have shown, these instruments and maneuvers are crucial in determining equilibrium royalties and may cause them to be high. Our interest here is how expected royalties affect investments and pre-investment licensing. We therefore abstract from how royalties are determined after litigation is filed, and summarize the results with the parameter $\rho \in [0, 1]$.

The game between the patentee and the developer is dynamic and of incomplete information, so we use the solution concept of perfect Bayesian equilibrium (PBE). Figure 1 illustrates the game.

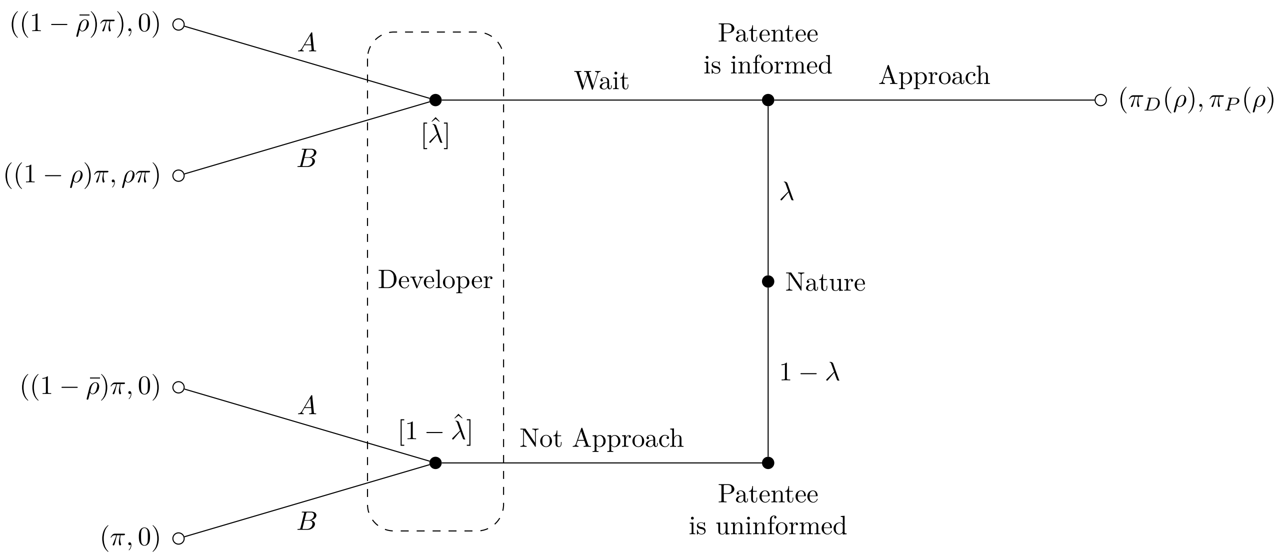


Figure 1: Description of the game

We start by deriving the payoffs from an ex ante licensing negotiation, when the patentee approaches before the developer chooses a technology.

Lemma 1. *If the patentee approaches ex ante, the Nash-barganing payoffs are*

$$\pi_P(\rho) = \begin{cases} \beta\bar{\rho}\pi & \text{if } \rho > \bar{\rho} \\ \rho\pi & \text{if } \rho \leq \bar{\rho} \end{cases}, \quad \pi_D(\rho) = \begin{cases} (1-\beta\bar{\rho})\pi & \text{if } \rho > \bar{\rho} \\ (1-\rho)\pi & \text{if } \rho \leq \bar{\rho} \end{cases}.$$

[9]Payoffs are the same as if settlement were possible, and in settling the firms avoided their own litigation costs and earned exactly their outside options gross of those costs.

Formally, a PBE specifies the developer's belief $\hat{\lambda}$, conditional on not being approached by the patentee ex ante, and the optimal actions for the patentee and the developer, conditional on $\hat{\lambda}$. The developer's strategy is to choose technology A with probability $a \in [0,1]$, and B with probability $1-a$. The patentee's strategy conditional on being informed is to approach with probability $\phi \in [0,1]$ and wait with probability $1-\phi$. The patentee's only action is 'Not Approach' conditional on being uninformed.

Taking the patentee's strategy $\phi \in [0,1]$ as given, the developer's belief is determined by Bayes rule so we must have

$$\hat{\lambda} = \frac{(1-\phi)\lambda}{(1-\phi)\lambda + 1 - \lambda} \tag{1}$$

Certain threat. Consider first the benchmark case of $\lambda = 1$. Implicitly, this captures situations where the developer has also searched and is as informed as the patentee.[10] It knows for sure there is a patentee that will claim patent infringement after B is chosen, i.e., $\hat{\lambda} = 1$.[11] We have the following result:

Proposition 1. *If $\lambda = 1$, the developer chooses B in equilibrium. For $\rho \leq \bar{\rho}$ there is multiplicity of equilibria, but they are payoff equivalent: the patentee approaches with probability $\phi^* \in [0,1]$ and receives $\rho\pi$. For $\rho > \bar{\rho}$, the patentee approaches for sure to negotiate a license ex ante.*

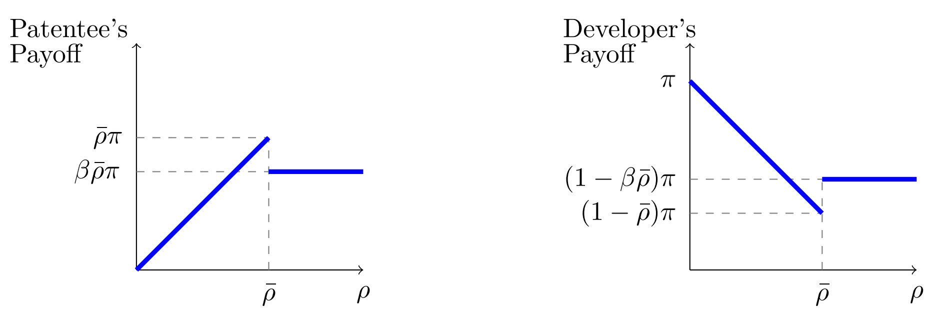


Figure 2: Payoffs for the developer and patentee for the case of a certain patentee ($\lambda = 1$). Although Proposition 1 shows that for $\rho \leq \bar{\rho}$ there are multiple equilibria, they are all payoff equivalent.

[10] Search is potentially valuable for the developer, because the optimal project choice may depend on whether there is a patentee. But note that it generates no value if the patentee does not search, because the developer would always be free to operate. Hence, in a more general model incorporating search directly, the two important settings (for project choice, negotiation and litigation) that emerge are: (1) the patentee and developer both search; and (2) only the patentee searches. Our model captures both, the former with the $\lambda = 1$ case and the latter with $\lambda \in (0,1)$.

[11] The $\lambda = 1$ case in our model is very similar to the "early negotiations" model of Lemley and Shapiro (2007, pp. 2003-05), where our developer's choice of project A is analogous to their downstream firm's option to redesign. They do not study private information.

Figure 2 shows the equilibrium payoffs when $\lambda = 1$. For both the patentee and the developer, the payoffs are non-monotonic in ρ when $\beta < 1$ (they are monotonic when $\beta = 1$). If $\rho \leq \bar{\rho}$, then payoffs fall with ρ for the developer firm and rise with ρ for the patentee. The developer's payoff declines with ρ because there is a patent for sure who will take a share ρ of the profit. However, in this range, the additional value created by technology B compensates the developer sufficiently so she 'ignores' the possibility of infringement.

For $\rho > \bar{\rho}$, payoffs are a constant function of ρ for both firms. In this range, the patentee is overly rewarded, so the developer prefers to implement the safe technology, A. But doing so reduces the joint surplus between the patentee and the developer: the patentee gets zero and the developer $(1 - \bar{\rho})\pi$. To prevent this surplus loss, the patentee approaches the developer and they negotiate for the use of technology B, so the joint surplus is π, and the patentee receives a share β of the increase in joint surplus, $\bar{\rho}\pi$. A key intuition is that higher ρ makes the developer's threat to use technology A credible, which improves the developer's bargaining position.

The patentee's equilibrium expected payoff never strictly exceeds $\bar{\rho}\pi$ in equilibrium. By our definition, the patentee never expects to be over-rewarded—i.e., never receives more than the marginal increase in the developer's profits from using technology B instead of technology A—when the developer knows for sure that there is a patentee.[12]

Uncertain threat. Consider now the case of $\lambda < 1$, so unless the patentee bargains ex ante, the developer may be at risk of patent infringement when choosing B.

Choosing technology A always give the developer a payoff of $(1-\bar{\rho})\pi$, regardless of the belief $\hat{\lambda}$. Choosing technology B, instead gives an expected payoff of $(1 - \hat{\lambda}\rho)\pi$. The developer's best response conditional on her belief is $a^*(\hat{\lambda}) = 1(\hat{\lambda}\rho > \bar{\rho}) + [0, 1] \cdot 1(\hat{\lambda}\rho = \bar{\rho})$. In words, the developer chooses A if $\hat{\lambda}\rho > \bar{\rho}$, B if $\hat{\lambda}\rho < \bar{\rho}$, and is indifferent between A and B when $\hat{\lambda}\rho = \bar{\rho}$.

The patentee's expected payoff from approaching with probability ϕ to negotiate a license ex ante is $\phi\pi_P(\rho) + (1 - \phi)(1 - a^*(\hat{\lambda}))\rho\pi$.

Proposition 2. *There are three non-generic cases depending on ρ:*[13]

1. *When $\rho \leq \bar{\rho}$ there are multiple equilibria. In all these equilibria, the developer chooses*

[12] Our definition of over-reward contrasts with the Lemley and Shapiro (2007) "no holdup" benchmark of $\theta\beta\bar{\rho}\pi$. As in their model, the patentee's equilibrium payoff with $\lambda = 1$ exceeds that benchmark for all cases except for when $\bar{\rho} = 0$ or when the patent is ironclad and $r = \beta$.

[13] For the generic case $\rho = \frac{\bar{\rho}}{\lambda}$, there are two equilibria. One is the limit case of the equilibria described in Proposition 2 part 2. The other one is similar to the part 3, except that project A is chosen with a probability greater than or equal to $\frac{\beta\bar{\rho}}{\rho}$.

B for sure and the patentee approaches with probability $\phi^* \in [0,1]$. *All these equilibria are payoff equivalent.*

2. *When* $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda}\right)$, *there is a unique equilibrium in which the developer chooses B for sure and the patentee waits for sure (ex post patent transfer).*

3. *When* $\rho > \frac{\bar{\rho}}{\lambda}$ *there is a unique equilibrium in which the developer chooses B with probability* $1 - a^* = \frac{\beta\bar{\rho}}{\rho}$ *and the patentee waits with probability* $1 - \phi^* = \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho-\bar{\rho})}$.

When $\rho \leq \bar{\rho}$ (Proposition 2, part 1), the result is essentially the same as Proposition 1. The developer will ignore the possibility of infringement and use technology B.

When $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda}\right)$ (Proposition 2, part 2) infringement risk makes a difference relative to the case $\lambda = 1$. Recall that if the developer was certain that a patentee will show up to claim infringement, the patentee approaches and negotiate ex ante whenever $\rho > \bar{\rho}$. When the risk is uncertain, $\lambda < 1$, the developer takes into account that, although the patentee is over-rewarded, it is possible that the patentee never shows up. In this range, choosing B for sure is worth it because the possibility that the patentee does not show up is large relative to over-rewarding the patentee. The patentee prefers to wait and hold up the developer once the (irreversible) technology choice has been made.

These cases of ex ante efficient investments and relatively common litigation is largely consisted with Lemley (2008), who notes that in many industries innovative firms generally ignore patents held by other firms. Specifically, technology companies generally instruct engineers not to read patents (Lemley 2008, p. 21), adopt policies of ignoring initial demand letters (Lemley 2008, p. 22), and choose not to substitute non-infringing redesigns (that are ready to go) until litigation is lost (p. 22). Litigation is indeed common, yet if the behavior is to be believed the threats had no effect on upfront investments.[14]

Note that when $\lambda < \bar{\rho}$, we cannot have $\rho > \frac{\bar{\rho}}{\lambda}$. Because the probability there is a patentee is less than the fraction of investment return that is specific to project B, the infringement threat is insufficient to force the developer to commit to the inferior technology in project A. Thus, the third case in Proposition 2 is feasible only $\lambda \in (\bar{\rho}, 1)$. When $\rho > \frac{\bar{\rho}}{\lambda}$ (Proposition 2, part 3), even accounting for the possibility that a patentee may never show up, the developer does not want to choose B with probability one. If the patentee shows up, the developer pays an excessively large compensation; it therefore prefers to use technology A. To prevent this choice, the patentee would like to approach the developer. But negotiating ex ante

[14]Lemley (2008, p. 21) notes that Intel, Cisco and Microsoft were routinely facing a collective 100 infringement suits at a time.

destroys the patentee's informational advantage and puts the developer in a strong barganing position. Therefore, in equilibrium, the patentee approaches with probability $\phi^* \in (0,1)$ to encourage the developer to adopt technology B while avoiding a negotiation in which the developer's bargaining position is strong. In this region, the unique equilibrium features mixed strategies: the patentee waits with probability $\frac{\bar{\rho}(1-\lambda)}{\lambda(\rho-\bar{\rho})}$ and the developer uses technology A with probability $\frac{\beta\bar{\rho}}{\rho}$.

Corollary 1. *For $\rho \leq \frac{\bar{\rho}}{\lambda}$, the technology choice is efficient, so the developer chooses B. For $\rho > \frac{\bar{\rho}}{\lambda}$, the equilibrium features the inferior technology, A, with probability $(1-\phi^*)a^*$, and the efficient technology, B, with probability*

$$1-(1-\phi^*)a^* = 1 - \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho-\bar{\rho})} \times \left(1 - \frac{\beta\bar{\rho}}{\rho}\right).$$

As ρ increases, the patentee is more likely to approach in equilibrium, i.e., ϕ^* increases in ρ. In contrast, the developer is more likely to chooses technology A if not approached, i.e., a^* increases in ρ. The inefficient technology choice is a consequence of the strategic delay in the negotiation by the patentee. Figure 3 shows the probability that the efficient technology choice results in equilibrium.

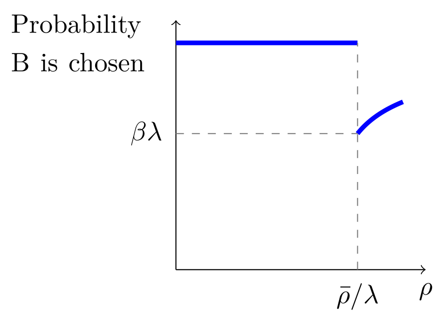


Figure 3: Probability that technology B is chosen in equilibrium.

The fact that the mixed-strategy equilibrium is unique has potential implications for interpreting real-world outcomes. The expected patentee payoff for $\rho \geq \frac{\bar{\rho}}{\lambda}$ quite clearly *does not* over-reward the patentee. But actual payoffs vary considerably, and the payoffs most likely to be observed in practice, those from litigation, *do* appear to over-reward the patentee. Indeed, any litigation payoff *must* over-reward the patentee that does not approach, because it does not always get the chance to sue. It expects to earn $\rho\pi$ if B is developed and it actually earns $r\pi$ if it wins a lawsuit; both are huge payoffs. In contrast, little notice is paid to zero payoffs that occur when choices like A are made, and payoffs from licensing (when the patentee approaches) are frequently kept private.

Proposition 3. *The equilibrium payoff the uninformed patentee is 0. The informed patentee's payoff is*

$$\pi_P^* = \begin{cases} \rho\pi & \text{if } \rho < \frac{\bar{\rho}}{\lambda}, \\ \beta\bar{\rho}\pi & \text{if } \rho < \frac{\bar{\rho}}{\lambda}. \end{cases} \tag{2}$$

The developer's payoff is

$$\pi_D^* = \begin{cases} (1-\lambda\rho)\pi & \text{if } \rho \leq \frac{\bar{\rho}}{\lambda}, \\ (1-\bar{\rho})\pi + \lambda\phi^*(1-\beta)\bar{\rho}\pi & \text{if } \rho < \frac{\bar{\rho}}{\lambda}, \end{cases} \tag{3}$$

Figure 4 shows the equilibrium payoffs for the patentee and for the developer. When $\beta = 1$, the payoffs are qualitatively similar to the payoffs in Figure 2. When $\beta < 1$, however, the developer's payoff increases in ρ. When ρ is large, the developer prefers to pay higher damages in case of infringement, because that increases the probability that an informed patentee approaches ex ante (ϕ^* increases).

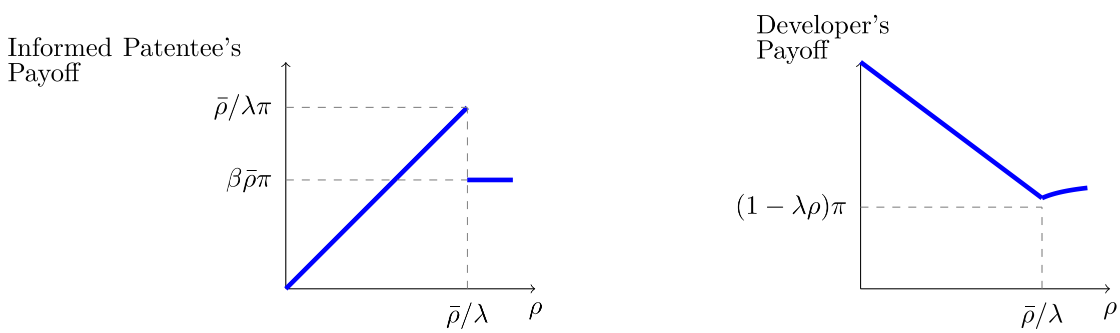


Figure 4: Ex ante expected equilibrium payoffs for the developer and the informed patentee for a moderately likely infringement threat, i.e., $\lambda \in (\bar{\rho}, 1)$.

3 Developer Search: Freedom to Operate

In the baseline model, the developer cannot take the initiative to find the patentee and negotiate a license ex ante. In practice, developers can choose to engage a "freedom to operate" (FTO) search before introducing a new technology. This helps to determine if there are any patents that are likely to be asserted against them in the future. In this section, we consider a FTO search stage in which the baseline game is the subgame that begins if the developer decides not to search.

The developer can conducts an FTO search at some cost K_D. The search is assumed to be "perfect" in the sense that, if there is a patent that would end up being enforced against the developer, then the search is guaranteed to find it. Therefore, the search will identify a patent threat with the probability λ, in which case the parties negotiate and the developer gets $\pi_D(\rho)$. With probability $1-\lambda$, the developer knows for sure that technology B is as safe as A, and therefore the payoff is π. Directly from Lemma 1 we have that the expected value of searching is given by

$$U_D^{FTO} = \begin{cases} (1-\lambda\rho)\pi, & \text{if } \rho \leq \bar{\rho} \\ (1-\beta\lambda\bar{\rho})\pi, & \text{if } \rho > \bar{\rho}. \end{cases} \tag{4}$$

When the developer searches and finds a patentee with a patent that could be a threat, the parties negotiate and there is ex ante licensing. If the developer does not find any threats, the developer uses B without any risk.

If the developer does not search, the payoff is the equilibrium payoff in Proposition 3. Thus, the marginal value of searching to the developer is $V_D^{\text{Search}} = U_D^{FTO} - \pi_D^*$, which is given by

$$V_D^{\text{Search}} = \begin{cases} 0, & \text{if } \rho \leq \bar{\rho} \\ (\rho - \beta\bar{\rho})\lambda\pi, & \text{if } \bar{\rho} < \rho \leq \bar{\rho}/\lambda \\ [1-\lambda(\phi^* + \beta - \beta\phi^*)]\bar{\rho}\pi, & \text{if } \rho > \bar{\rho}/\lambda. \end{cases} \tag{5}$$

Figure 5 shows the value of searching for the developer, showing that incentives are non-monotonic. The maximum value of searching is $(1-\lambda\beta)\bar{\rho}\pi$ attained at $\rho = \bar{\rho}/\lambda$. When ρ is sufficiently low, a FTO search provides no value. This is because the only benefit of searching is to avoid paying excessive fees ($\rho > \bar{\rho}$) due to delayed enforcement. An implication is that when patent quality is low (θ is low), the developer has no incentive to search. This is consistent with Lemley (2008), who argued that developers routinely "ignore patents," i.e. they do not attempt to search for them before launching a new technology, even if their is a sizeable probability of getting sued by a PAE. Second, the value of search is nonmonotonic for parameter values $\rho > \bar{\rho}$. The reason is that for $\rho > \bar{\rho}/\lambda$ the patentee approaches the developer with some probability (in contrast to the case $\rho \in (\bar{\rho}, \bar{\rho}/\lambda)$ when the patentee always waits). Thus, the value of searching decreases.

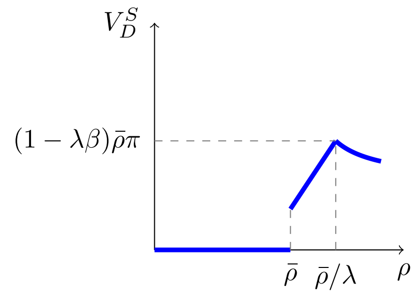


Figure 5: Developer's value of conducting a 'Freedom to Operate' search.

Proposition 4. *A developer performs a 'Freedom to Operate' search only if* $K_D \leq (1-\lambda\beta)\bar{\rho}\pi$. *In this case, the search is performed if and only if* $\rho \in (\rho_S^L, \rho_S^R)$ *where* $\bar{\rho} \leq \rho_S^L < \rho_S^R \leq 1$.

Proof. Direct from the text. □

4 Patentee might Arrive Late

In our baseline model, the patentee was either informed before the developer made a choice or it was uninformed. In practice, patentees can be informed ex-post about a developer infringing on its patente. To accommodate for this case, let there be three types of patentees: ex ante informed (type I), uninformed (type U), and ex post informed (type E). Suppose that it is common knowledge that the frequency of these types are λ_I, λ_U, λ_E, respectively. The difference with the baseline model is type E: This type of patentee discover infringement after the technology choice was made, and therefore its only option to monetize its patent is to threaten to sue. Thus, the patentee of type E's payoff is $\rho\pi$, and the developer gets $(1-\rho)\pi$ if chooses B and faces a patentee of type E.

The developer's information set where she does not meet the patentee before choosing a technology has three nodes, one for the patentee type. The developer forms beliefs over these nodes, which we call $\hat{\lambda}_I$, $\hat{\lambda}_U$, and $\hat{\lambda}_E$. If an informed patentee approaches the developer with probability ϕ, then Bayes' rule implies that the developer beliefs are

$$\hat{\lambda}_I = \frac{\lambda_I(1-\phi)}{\lambda_I(1-\phi)+\lambda_E+\lambda_U},$$

$$\hat{\lambda}_E = \frac{\lambda_E}{\lambda_I(1-\phi)+\lambda_E+\lambda_U},$$

and $\hat{\lambda}_U = 1 - \hat{\lambda}_I - \hat{\lambda}_E$. The developer's payoff from choosing A is the same as in the baseline case. The developer's payoff from choosing B, however, is

$$(\hat{\lambda}_I + \hat{\lambda}_E)(1-\rho)\pi + (1 - (\hat{\lambda}_I + \hat{\lambda}_E))\pi = (1 - (\hat{\lambda}_I + \hat{\lambda}_E)\rho)\pi.$$

The developer knows that she will be liable for damages after choosing B when she faces a patentee of type I or E. Let $\hat{\lambda} = \hat{\lambda}_I + \hat{\lambda}_E$. $a^*(\hat{\lambda}) = 1(\hat{\lambda}\rho > \bar{\rho}) + [0,1] \cdot 1(\hat{\lambda}\rho = \bar{\rho})$. In words, the developer chooses A if $\hat{\lambda}\rho \geq \bar{\rho}$, B if $\hat{\lambda}\rho < \bar{\rho}$, and is indifferent between A and B when $\hat{\lambda}\rho = \bar{\rho}$.

The informed patentee's expected payoff from approaching with probability ϕ to negotiate a license ex ante is $\phi\pi_P(\rho) + (1-\phi)(1-a^*(\hat{\lambda}))\rho\pi$.

Proposition 5. *There are three non-generic cases depending on ρ:*

1. *When $\rho \leq \bar{\rho}$ there are multiple equilibria. In all these equilibria, the developer chooses B for sure and the patentee approaches with probability $\phi^* \in [0,1]$. All these equilibria are payoff equivalent.*
2. *When $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda_I + \lambda_E}\right)$, there is a unique equilibrium in which the developer chooses B for sure and the patentee waits for sure (ex post patent transfer).*
3. *When $\rho \in \left(\frac{\bar{\rho}}{\lambda_I+\lambda_E}, \frac{\bar{\rho}(1-\lambda_I)}{\lambda_E}\right)$ there is a unique equilibrium in which the developer chooses B with probability $a^* = \frac{\beta\bar{\rho}}{\rho}$ and the patentee waits with probability $1 - \phi^* = \frac{\bar{\rho}(1-\lambda_I) - \rho\lambda_E}{\lambda_I(\rho - \bar{\rho})}$.*
4. *When $\rho > \frac{\bar{\rho}(1-\lambda_I)}{\lambda_E}$ there an* inefficient *equilibrium in which the developer chooses A for sure and the patentee approaches to license ex ante whenever informed.*

Note that when $\lambda_E \to 0$, Proposition 5 collapses to Proposition 2. When $\lambda_E > 0$, Proposition 5 (part 4) shows that it is possible that the developer invests in A *even if* the patentee has no intention to strategically wait. The reason is that the patentee gets over compensated (they receive more than $\bar{\rho}$), but it does not find the developer in time to prevent her from choosing the inefficient technology.

Proposition 6. *For any combination (λ_I, λ_E) such that $\lambda_I + \lambda_E = \lambda$, the patentee's payoff (weakly) decreases in λ_E.*

Proposition 6 shows that the patentee's best scenario is to find potential infringers as soon as possible (i.e., $\lambda_I = \lambda$, as in the baseline case). This is not surprising as it creates a strategic advantage for the patentee that it is lost whenever the patentee finds infringers later on. Finding a potential infringer early on does not mean that the patentee will always negotiate a license ex ante, but instead that the patentee will strategically choose when to approach. Thus, any market friction that prevent the patentee from finding developers early on—even if they do not approach them—increases the chances that in equilibrium the developer picks the inferior technology.

The developer's incentives to search are qualitatively similar to the baseline case except when $\rho > \frac{\bar{\rho}(1-\lambda_I)}{\lambda_E}$. In this case, if the developer does not search is always going to pick project A while searching can leads to project B being chosen. The expected value of searching is constant and equal to $(1-\beta\lambda\bar{\rho})\pi - (1-\bar{\rho})\pi = (1-\beta\lambda)\pi\bar{\rho}$. In contrast to the baseline case, when $\lambda_E > 0$, the patentee will search for high values of ρ.

5 Practicing Patentee

Return to the baseline model with all early-arriving patentees, but now assume that the patentees are competitors. If the developer chooses project B, the patentee earns $w - h\pi$ gross of any royalty or other payments. If the developer chooses project A, the patentee earns w. Let $h > \bar{\rho}$, so that the total payoffs are highest when project A is chosen. This reflects the usual efficiency effect of monopoly.

It remains the case that conditional on the developer's belief $\hat{\lambda}$, a developer that is not approached will choose project B iff $\rho \leq \frac{\bar{\rho}}{\hat{\lambda}}$. Many other aspects of the analysis change, however. Now, if the patentee approaches, the Pareto optimal project is project A. Hence any approach leads to project A being chosen.

The payoffs depend upon outside options. If the developer's outside option is to choose project B ($\rho \leq \bar{\rho}$), then the bargaining surplus is $(h-\bar{\rho})\pi$. The patentee earns

$$\pi_P(\rho) = w - (h-\rho)\pi + \beta(h-\bar{\rho})\pi \tag{6}$$

while the developer earns

$$\pi_D(\rho) = \pi - \rho\pi + (1-\beta)(h-\bar{\rho})\pi \tag{7}$$

Note that this bargain is essentially anticompetitive. The developer is being paid not to use the patented technology.

If on the other hand the developer's outside option is to choose project A ($\rho > \bar{\rho}$), then there is no bargaining surplus, and $\pi_P(\rho) = w$ and $\pi_D(\rho) = \pi(1 - \bar{\rho})$. This is essentially a null bargain.

For $\rho < \bar{\rho}$, the unique equilibrium is that the patentee approaches and the anticompetitive bargain obtains. Note that because $\rho < \bar{\rho}$,, it is not possible for $\rho > h$. Hence, with low ρ it is not possible for the patentee to gain if there is no bargain and the developer infringes. Now, if $\rho > \bar{\rho}$, then it is possible to have $\rho > h$, in which case the patentee earns such high royalties that conditional on waiting it would prefer that the developer choose project B.

For $\rho > \bar{\rho}$, if the patentee approaches, then the developer's outside option (with belief $\hat{\lambda} = 1$) now becomes project A. Hence, in equilibria where the patentee approaches, the null bargain obtains.

It remains to determine when the patentee approaches and what the developer does absent an approach. If $\rho \in (\bar{\rho}, h)$, then the equilibrium is for the patentee to approach always, and for the developer to choose project B if not approached. The patentee cannot gain by waiting because $\rho < h$, and the developer has nothing to fear if it is not approached. It updates to $\hat{\lambda} = 0$ if there is no approach.

If $\rho > h$, then there are two cases. If $\rho \in [h, \frac{\bar{\rho}}{\lambda}]$, then the unique equilibrium is for the patentee to wait and the developer to pick B. In this case, the patentee is over-rewarded in an infringement suit so it prefers to wait. The developer is not very worried about a threat so it picks B. This is the only scenario with late lawsuits.

If $\rho > Max\{\frac{\bar{\rho}}{\lambda}, h\}$, then the unique equilibrium is for the patentee to wait and for the developer to choose A if not approached. These are clearly best responses. At first glance it may seem surprising that there is not an alternative equilibrium. But note that if the patentee always approaches, then a developer who is not approached would update $\hat{\lambda} = 0$ and would be better off picking B if not approached. But then the patentee would prefer to wait. Any mixing between the developer ($a^* < 1$) cannot make the patentee indifferent, as an approach would yield the null bargain which is the same payoff as when the patentee waits and the developer chooses A. Any chance that the developer picks B would lead to the developer always wanting to wait.

Hence, we get patentee approaching as part of a unique equilibrium for any $\rho < h$, and patentee waiting as part of a unique equilibrium for any $\rho > h$.

Summary of equilibria:

The first term in the {,} below is the patentee's choice of Approach or Wait. The second is the developer's choice conditional on no approach happening.

1. $\rho < h$:{Approach,B}

a. $\rho < \bar{\rho}$: "anticompetitive" settlement obtains.
b. $\bar{\rho} < \rho < Min\{h, \frac{\bar{\rho}}{\lambda}\}$: null bargain obtains.
c. $\bar{\rho} < \frac{\bar{\rho}}{\lambda} < \rho < h$: null bargain obtains.

2. $\rho > h$:

a. $\bar{\rho} < h < \rho < \frac{\bar{\rho}}{\lambda}$: {Wait,B}, late litigation obtains, troll-like outcome.
b. $\bar{\rho} < Max\{h, \frac{\bar{\rho}}{\lambda}\} < \rho$: {Wait,A}, no real interaction.

Hence, in our model, practicing patentees are more likely to approach. When $\rho < \bar{\rho}$, practicing patentees have strict incentives to approach whereas non-practicing patentees are indifferent. When $\rho \in [\bar{\rho}, h)$, practicing patentees have strict incentives to approach while non-practicing patentees have strict incentives to wait. When $\rho > h$, there is a case where late litigation occurs with practicing patentees, but late litigation also occurs with non-practicing patentees. For excessively over-rewarded patentees ($\rho > \frac{\bar{\rho}}{\lambda}$) there are cases (2b) where practicing patentees wait, but this never leads to litigation. For such high ρ, non-practicing patentees sometimes do file late lawsuits.

6 Ex Ante Innovation and Technology Transfer

Suppose that by paying a fixed cost πK, the developer develops alternative B with probability $\alpha \in (0, 1)$. If successful, the developer chooses B for sure. Under belief $\hat{\lambda}$, the developer finds worth paying the fixed cost when $\alpha(1 - \hat{\lambda}\rho)\pi - \pi K > (1 - \bar{\rho})\pi$ or, equivalently,

$$\bar{\rho} - \alpha\hat{\lambda}\rho \geq K + 1 - \alpha. \tag{8}$$

Proposition 7. *There are three non-generic cases depending on ρ:*

1. *When $\rho \leq \max\left\{\frac{\bar{\rho}-K-1+\alpha}{\alpha}, \frac{\beta\bar{\rho}}{\alpha}\right\}$ the equilibrium is efficient: the patentee approaches for sure (ex ante licensing) and the developer chooses B for sure.*

2. *When* $\rho \in \left(\max\left\{\frac{\bar{\rho}-K-1+\alpha}{\alpha}, \frac{\beta\bar{\rho}}{\alpha}\right\}, \bar{\rho} - K - 1 + \alpha\alpha\lambda\right)$*, there is a unique (inefficient) equilibrium in which the developer attempts to develop B, and succeeds with probability* α*, and the patentee waits for sure (ex post patent transfer).*

3. *When* $\rho > \max\left\{\frac{\bar{\rho}-K-1+\alpha}{\alpha\lambda}, \frac{\beta\bar{\rho}}{\alpha}\right\}$ *there is a unique equilibrium in which the developer attempts to develop B with probability* $1 - a^* = \frac{\beta\bar{\rho}}{\rho}$ *and the patentee approaches with probability* $\phi^* = 1 - 1 - \frac{(\bar{\rho}-K-1+\alpha)(1-\lambda)}{\lambda(1-\alpha+\alpha\rho-\bar{\rho}+K)}$.

The figures below compare the equilibria in different regions in the baseline case and in the extension with fixed re-discovery cost K, and success probability α.

Figure 6: Baseline case

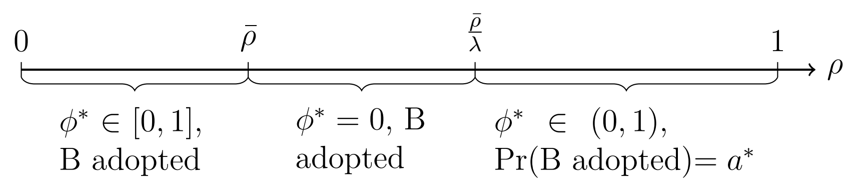
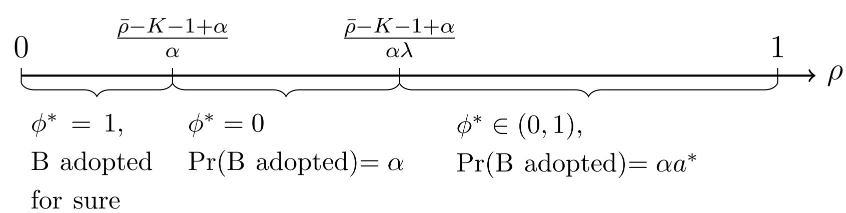


Figure 7: Costly Development when $\bar{\rho} > \frac{K+1-\alpha}{1-\lambda\beta}$.



We obtain similar results in a model where investment is a continuous choice. Specifically, to succeed with probability α, the patentee must invest an amount $c(\alpha)$, defined by

$$c(\alpha) = \frac{1}{2}k\pi\alpha^2,$$

where $k > 0$ is a cost parameter.

Proposition 8. *An equilibrium exists for all parameter values. In all of them, the developer will invest in technology B if he is not approached (and he never mixes when* $k > 0$*).*

1. *The patentee approaches in equilibrium* ($\phi^* = 1$) *if either (a)* $\rho \leq \bar{\rho}$; *or (b)* $\rho > \bar{\rho}$ *and* $\alpha^*(\lambda)\rho < \alpha^*(0)\rho \leq \beta\bar{\rho}$.[15]

2. *The patentee waits in equilibrium* ($\phi^* = 0$) *if* $\rho > \bar{\rho}$ *and either (a)* $\alpha^*(\lambda) = 1$; *or (b)* $\beta\bar{\rho} \leq \alpha^*(\lambda)\rho < \alpha^*(0)\rho$.

3. *The patentee mixes in equilibrium* ($0 < \phi^* < 1$) *if* $\rho > \bar{\rho}$ *and* $\alpha^*(\lambda)\rho < \beta\bar{\rho} < \alpha^*(0)\rho$.

7 Empirical Exercise

We collect patent infringement lawsuits from the Stanford NPE Litigation Database and we complement this information with data from Lex Machina. For each lawsuit, we observe the filing date, the identity and type (NPE or PE) of the patent asserter, the identity of the alleged infringer, the case resolution, and all the patents involved in the lawsuit. We obtained information on patent characteristics from the USPTO.

We focus on lawsuits filed between 2014 and 2020. Figure 8 plots the average age of patents in lawsuits between 2014 and 2020. The figure shows that practicing entities, on average, litigate younger patents.

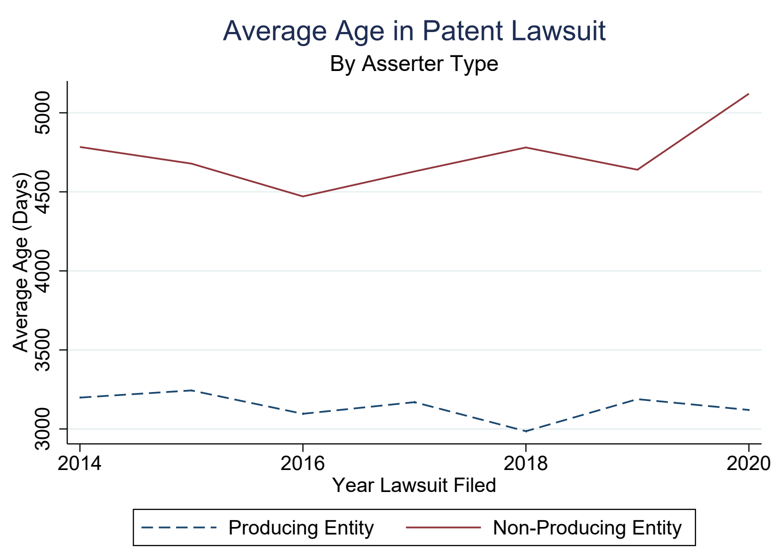


Figure 8: Average patent age in lawsuits filed between 2014-2020 by asserter type (practicing or non-practicing entity).

[15] In the case $\rho \leq \bar{\rho}$, there can also be mixed equilibria $\phi^* \in (0, 1)$, but only if $\alpha^*(\hat{\lambda}(\phi^*)) = 1$.

Until March 21, 2017, defendants accused of patent infringement could use the "laches" defense and claim that the patentee took too long to file an infringement lawsuit. Moreover, the delay was presumed to be unreasonable if the patentee had waited more than six years after having knowledge of the alleged infringement. On March 21, 2017, however, the Supreme Court issued its decision in *SCA Hygiene Products Aktiebolag v. First Quality Baby Products, LLC*, which established that "laches" is no longer a valid defense for infringement. This ruling means that patentees have weaker incentives to file lawsuits quickly since delayed enforcement cannot be counteracted by invoking the laches defense.

We empirically investigate the impact of removing the laches defense on delayed enforcement. Presumably, the removal of the laches defense pushes all patent asserters to use "older" patents but it may have disproportionately benefited non-producing entities, which typically assert older patents (see Figure 8). To this end, we run several specifications of the following model:

$$Y_{it} = \alpha PE_i + \beta PE_i \times \text{Post-Laches}_{it} + \gamma Q_i + \varepsilon_{it}.$$

Y_{it} is an outcome of interest for lawsuit i filed at year t. PE_i is an indicator that takes the value 1 if lawsuit i was filed by a producing entity, and zero if the asserter was a non-producing entity. The variable Post-Laches is an indicator that takes the value of 1 if lawsuit i was filed after March 21, 2017. The variable $Q_i \in [-1, 1]$ is the average patent quality of lawsuit i, which we compute using a latent variable model explain in subsection 7.1.

Table 1: Impact of Laches on Patent Lawsuits

	(1) Mean(Age)	(2) Mean(Age)	(3) Bundle Size	(4) Min(Age)	(5) Grant Delay
Producing Entity (PE)	-1011.1***	-1013.6***	0.797***	-1391.8***	-1235.3***
	(38.39)	(37.74)	(0.0704)	(39.21)	(35.23)
Post-Laches	243.3***	193.9***	0.0917	184.3***	-1075.4***
	(34.58)	(34.00)	(0.0542)	(39.33)	(34.11)
PE × Post-Laches	-307.8***	-286.5***	0.249**	-271.2***	-228.5***
	(47.51)	(46.51)	(0.0857)	(51.90)	(47.46)
Quality Index		-2001.2***	1.082***	-2351.3***	-2172.5***
		(57.66)	(0.0710)	(60.18)	(58.27)
N	27706	27706	27706	27727	27727
R^2	0.177	0.212	0.049	0.188	0.252

Notes: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. All the regressions control for litigation venue and NBER patent classification. Standard errors are robustly estimated.

Table 1 shows the differential effect of laches on producing and non-producing entities. Column 1 shows that producing entities systematically use younger patents than non-practicing entities: on average, around 1011 days (2 years 9 months) younger. Post-laches producing entities used slighlty younger patents while non-producing entities began using older patents: compared to practicing entities, NPE use patents that are around 10 months (307.8 days) older, on average, after the laches defense was removed. Column 2 shows that the effects are similar when we control for the average quality of the patent involved in the lawsuit. The coefficient on Quality shows that higher the quality are litigated sooner than lower-quality patents.

Table 1 (Column 3) uses the number of patents in the lawsuit as the outcome variable of interest. The estimates show that producing entities use more patents when they file a lawsuit. Laches did not have an impact on the number of patents in the lawsuit of non-practicing entities, but practicing entities slightly increased the number of patents involved in each lawsuit.

Table 1 (Column 4) is a robustness check using the minimum age, rather than the average age of the patents in the lawsuit. The results are similar to those in Column 1.

Lastly, Column 5 uses the average USPTO grant delay as the outcome variable of interest. Compared to practicing entities, non-producing entities use patents that had much longer prosecution: on average 3 years and 4 months longer. Post-laches, both practicing and non-practicing entities use patents that took about 3 years longer to prosecute. However, non-practicing entities used patents that took even longer to prosecute.

Table 2 estimates different model specifications controlling for the identity of the asserter/infringer. Column 1 controls only for venue. Column 2 includes a venue and asserter fixed effects. Column 3 includes a venue and infringer fixed effects. Column 4 includes a venue, asserter and infringer fixed effects, and Column 5 includes additionally a year fixed effect. Though the magnitude of the effect changes, the direction of the coefficient indicates that laches had a differential implication for NPEs and PEs.

Table 2: Impact of Fixed effects on Average Age of Patent in Lawsuits

	(1) Age	(2) Age	(3) Age	(4) Age	(5) Age
Producing Entity (PE)	-1324.6***	272.7***	-587.9***	432.1**	422.6**
	(35.31)	(76.58)	(84.85)	(150.7)	(146.5)
Post-Laches	217.3***	506.4***	363.6***	500.8***	173.9*
	(34.95)	(37.91)	(61.59)	(59.16)	(83.31)
PE × Post-Laches	-285.4***	-170.4**	-527.0***	-338.6**	-376.8***
	(48.32)	(57.35)	(94.63)	(103.1)	(102.4)
N	27734	23268	13812	11886	11886
R^2	0.146	0.837	0.439	0.887	0.888

Notes: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. All the regressions control for litigation venue. Standard errors are robustly estimated.

7.1 Patent Quality Index

We collected all 6,596,989 U.S. granted patents from 1990 to 2020. For each patent, we observe its filing date, grant date, type (utility, plant, design), number of claims, number of independent claims, cumulative forward citations after 5 years from the grant date, indicator for 4 year maintenance fee paid, NBER classification, International Patent Classification (IPC), technology center, examiner, cumulative number of lawsuits after 3 years from the grant date.

Similar to what other papers in the literature have done (e.g. Lanjouw and Schankerman, 2004), we estimate an exogenous latent variable model. We use seven observable variables, $(Y_j)_{j=1}^{7}$, that are correlated with the unobserved patent quality:

$$\mathrm{Y}_{ji} = \alpha_j + \lambda_j q + \varepsilon_{ij}.$$

Y corresponds to claims, independent claims, cumulative forward citations after 5 years, litigation, number of litigation, maintenance fee, delay from filing to approval.

We estimate the model separately for seven categories: Each one of the six NBER classifications plus one category for design patents. We only have NBER classifications for patents granted before 2015. We use a multinomial logit to predict the NBER classifications for patents granted between 2015-2020 using data on technology centers and IPC classifications from 2010-2015.[16] Design patents do not have claims nor they need to be renewed, so we only use four categories for design patents.

After we estimate the model, we predict an index of quality for each patent using empirical Bayes means prediction.

8 References

Lemley, Mark A and Carl Shapiro (2006) "Patent holdup and royalty stacking," *Tex. L. Rev.*, Vol. 85, p. 1991.

[16] The multinomial logit model correctly predicts the NBER classification in around 80 percent of the cases.

A Proofs

Proof of Lemma 1

Proof. When the patentee approaches the developer before the technology choice has been made, the developer can use the threat to use technology A in the licensing negotiation. This is threat is credible only when the technology A's payoff is larger than the payoff of using technology B, infringing, and settling the lawsuit. If indifferent, we assume that the developer always chooses A. Therefore, using technology A is a credible threat if and only if

$$(1-\bar{\rho})\pi \geq (1-\rho)\pi \Leftrightarrow \rho \geq \bar{\rho}.$$

When $\rho \geq \bar{\rho}$, disagreement reduces the joint surplus by $\pi - (1-\bar{\rho})\pi = \bar{\rho}\pi$ because the developer will use the inferior technology, A. Under Nash bargaining, the patentee gets a share β of this surplus and the developer the remaining share. When $\rho > \bar{\rho}$ the developer does not have a credible threat—the developer will choose technology B regardless. Thus, the ex ante negotiation prevents a lawsuit (which cost is normalized to zero) and the patentee gets $\rho\pi$. □

Proof of Proposition 1

Proof. The developer chooses B if $\rho < \bar{\rho}$, chooses A if $\rho > \bar{\rho}$ and is indifferent if $\rho = \bar{\rho}$. The patentee is indifferent about approaching and not approaching when $\rho < \bar{\rho}$, as there is no bargaining surplus. Therefore, for $\rho < \bar{B}$ there is multiplicity of equilibria: the developer chooses B for sure and the patentee approaches with probability $\phi^* \in [0,1]$. When $\rho > \bar{\rho}$, the patent holder approaches for sure, as otherwise it would earn a payoff of zero. When $\rho = \bar{\rho}$, the developer mixes between A and B. Let A be chosen by the developer with probability a^*. By approaching with probability ϕ, the patentee gets $(1-\phi)(1-a^*)\rho\pi + \phi\rho\pi$. Thus, in equilibrium, $\phi = 1$ when $a^* > 0$. □

Proof of Proposition 2

Proof. We have three cases:

(a) Separating equilibrium, $\phi^* = 1$. When an informed patentee always approach, we have

$\hat{\lambda} = 0$, and therefore the developer chooses B. The patent holder's payoff from deviating and choosing to approach with a different probability ϕ is $\rho\pi + \phi(\pi_P(\rho) - \rho\pi)$. We can sustain $\phi^* = 1$ in equilibrium only if $\pi_P(\rho) \geq \rho\pi$. When $\rho \leq \bar{\rho}$, this condition holds with equality. When $\rho > \bar{\rho}$, this condition is $\beta\bar{\rho}\pi \geq \rho\pi$, or $\beta\bar{\rho} \geq \rho$, which cannot hold since it would imply that $\beta\bar{\rho} \geq \rho > \bar{\rho}$.

(b) Pooling equilibrium, $\phi^* = 0$. When the patentee waits for sure, we have $\hat{\lambda} = \lambda$. Under this belief, the developer chooses A whenever $\lambda\rho > \bar{\rho}$. If this condition holds, the informed patentee receives zero, so it would prefer to deviate to 'Approach.' Thus, for $\phi^* = 0$ to be an equilibrium, it must be that $\lambda\rho \leq \bar{\rho}$. If $\lambda\rho < \bar{\rho}$, the developer chooses B, and the patentee gets $\rho\pi$ by not approaching. The patentee must no have incentives to deviate, which happens when $\rho\pi \geq \pi_P(\rho)$. This condition holds with equality for $\rho \leq \bar{\rho}$ and it also holds for $\rho > \bar{\rho}$ because $\rho > \bar{\rho} \Rightarrow \rho \geq \beta\bar{\rho}$. Thus, when $\lambda\rho < \bar{\rho}$, $\phi^* = 0$ is an equilibrium. Lastly, when $\lambda\rho = \bar{\rho}$, the developer is indifferent between A and B, so suppose the developer chooses A with probability $a^* \in [0, 1]$. In that case, the patentee's expected payoff from waiting is $(1 - a^*)\rho\pi$, while the payoff from approaching is $\pi_P(\rho)$. For $\rho \leq \bar{\rho}$, the patentee is indifferent when $a^* = 0$. For $\rho > \bar{\rho}$, the patentee prefers to wait iff $(1 - a^*)\rho\pi \geq \beta\bar{\rho}\pi$ or, equivalently, $a^* \leq 1 - \frac{\beta\bar{\rho}}{\rho}$. Thus, $\phi^* = 0$ is an equilibrium whenever $\lambda\rho \leq \bar{\rho}$.

(c) Semi-pooling, $\phi^* \in (0, 1)$: For the patentee to mix between waiting and approaching, we must have $\pi_P(\rho) = (1 - a^*(\hat{\lambda}))\rho\pi$. We study two cases:

(c.1) When $\rho > \bar{\rho}$, we need $\beta\bar{\rho} = (1 - a^*(\hat{\lambda}))\rho$. Since $\beta \leq 1$, this can hold only if $a^*(\hat{\lambda}) \in (0, 1)$, meaning that the developer is indifferent between A and B, i.e., when $\hat{\lambda}\rho = \bar{\rho}$. This requires

$$\frac{(1 - \phi)\lambda}{(1 - \phi)\lambda + 1 - \lambda} = \frac{\bar{\rho}}{\rho} \Leftrightarrow \phi^* = 1 - \frac{\bar{\rho}(1 - \lambda)}{\lambda(\rho - \bar{\rho})}.$$

Clearly $\phi^* < 1$. However, $\phi^* > 0$ if and only if $\lambda\rho > \bar{\rho}$.

(c.2) When $\rho < \bar{\rho}$, then $\hat{\lambda}\rho \leq \rho < \bar{\rho}$. Therefore, $a^*(\hat{\lambda}) = 0$ and $\pi_P(\rho) = \rho\pi$. So in this case, any $\phi^* \in (0, 1)$ will be an equilibrium.

(c.3) When $\rho = \bar{\rho}$, $\pi_P(\rho) = \rho\pi$. For this to be an equilibrium, the developer must pick B with positive probability and leave the patentee indifferent between approaching or waiting. So we need $\hat{\lambda}\rho \leq \bar{\rho}$, in which case the patent holder gets $(1 - a^*(\hat{\lambda}))\rho\pi$ from waiting. In order for the patentee to not deviate, we must have $a^*(\hat{\lambda}) = 0$, i.e., when indifferent, the developer must choose B. This is always an equilibrium because $\hat{\lambda} \leq 1$.

□

Proof of Proposition 3

Proof. **Equilibrium payoffs.** The equilibrium payoff the uninformed patentee is 0. The informed patentee receives a payoff of $\rho\pi$ when $\rho < \frac{\bar{\rho}}{\lambda}$. When $\rho > \frac{\bar{\rho}}{\lambda}$, the informed patentee is indifferent between waiting and approaching, so her equilibrium payoff is $\beta\bar{\rho}\pi$.

When $\rho < \frac{\bar{\rho}}{\lambda}$, the developer always chooses B and her expected payoff is $\lambda(1-\rho)\pi + (1-\lambda)\pi$. When $\rho > \frac{\bar{\rho}}{\lambda}$ both the patentee and the developer play mixed strategies. The developer's is indifferent between A and B if the patentee does not approach, which happens with probability $1 - \lambda\phi^*$, so her expected equilibrium payoff is $(1-\lambda\phi^*)(1-\bar{\rho})\pi + \lambda\phi^*(1-\beta\bar{\rho})\pi$ which equals $(1-\bar{\rho})\pi + \lambda\phi^*(1-\beta)\bar{\rho}\pi$, where $\phi^* = 1 - \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho-\bar{\rho})}$. □

Proof of Proposition 5

Proof. We study three cases:

1. Equilibria with $\phi^* = 1$. In contrast to the baseline case, there is residual risk even if informed patentees always approach ex ante. So, $\hat{\lambda}_I = 0$, $\hat{\lambda}_E = \frac{\lambda_E}{\lambda_E+\lambda_U}$, and $\hat{\lambda}_U = \frac{\lambda_U}{\lambda_E+\lambda_U}$.

   Thus, the developer chooses A whenever $\hat{\lambda}_E\rho > \bar{\rho}$. In the case, the patentee does not have a profitable deviation, so there is inefficiency in equilibrium.

   Thus, the developer chooses B whenever $\hat{\lambda}_E\rho < \bar{\rho}$. In this case, the developer compares $\rho\pi$ with $\pi_D(\rho)$. Whenever $\rho \leq \bar{\rho}$, the developer is indifferent, so this is an equilibrium. However, when $\hat{\lambda}_E\rho < \bar{\rho} < \rho$, the developer always want to deviate because $\beta\bar{\rho}\pi < \rho\pi$.

   Thus, there is an equilibrium in which $\phi^* = 1$ if and only if $\frac{\lambda_E}{\lambda_E+\lambda_U}\rho > \bar{\rho}$, or $\frac{\lambda_E}{\lambda_E+\lambda_U}\rho < \bar{\rho}$ and $\rho \leq \bar{\rho}$.

2. Equilibria with $\phi^* = 0$. Bayes implies $\hat{\lambda}_I = \lambda_I$, $\hat{\lambda}_E = \lambda_E$, and $\hat{\lambda}_U = 1 - \hat{\lambda}_I - \hat{\lambda}_E$. Let $\lambda = \lambda_I + \lambda_E$. From here, the proof is identical to the proof of Proposition 2, so $\phi^* = 0$ and B is chosen by the developer conditional on no approach whenever $\lambda\rho \leq \bar{\rho}$.

3. Semi-pooling, $\phi^* \in (0,1)$: Define $\hat{\lambda} = \hat{\lambda}_I + \hat{\lambda}_E$. For the patentee to mix between waiting and approaching, we must have $\pi_P(\rho) = (1 - a^*(\hat{\lambda}))\rho\pi$. We study two cases:

(c.1) When $\rho > \bar{\rho}$, we need $\beta\bar{\rho} = (1 - a^*(\hat{\lambda}))\rho$. Since $\beta \leq 1$, this can hold only if $a^*(\hat{\lambda}) \in (0, 1)$, meaning that the developer is indifferent between A and B, i.e., when $\hat{\lambda}\rho = \bar{\rho}$. This requires

$$\frac{\lambda_I(1-\phi) + \lambda_E}{\lambda_I(1-\phi) + \lambda_E + \lambda_U} = \frac{\bar{\rho}}{\rho} \Leftrightarrow \phi^* = 1 - \frac{\bar{\rho}(1-\lambda_I) - \rho\lambda_E}{\lambda_I(\rho - \bar{\rho})}.$$

For this probability to be well defined we need $(\lambda_I + \lambda_E)\rho \geq \bar{\rho}$ and in addition we require $\lambda_E\rho \leq \bar{\rho}(1 - \lambda_I)$.

(c.2) When $\rho < \bar{\rho}$, then $\hat{\lambda}\rho \leq \rho < \bar{\rho}$. Therefore, $a^*(\hat{\lambda}) = 0$ and $\pi_P(\rho) = \rho\pi$. So in this case, any $\phi^* \in (0, 1)$ will be an equilibrium.

(c.3) When $\rho = \bar{\rho}$, $\pi_P(\rho) = \rho\pi$. For this to be an equilibrium, the developer must pick B with positive probability and leave the patentee indifferent between approaching or waiting. So we need $\hat{\lambda}\rho \leq \bar{\rho}$, in which case the patent holder gets $(1 - a^*(\hat{\lambda}))\rho\pi$ from waiting. In order for the patentee to not deviate, we must have $a^*(\hat{\lambda}) = 0$, i.e., when indifferent, the developer must choose B. This is always an equilibrium because $\hat{\lambda} \leq 1$.

□

Proof of Proposition 6

Proof. The patentee's payoff is independent of the distribution (λ_I, λ_E) whenever $\rho \leq \frac{\bar{\rho}}{\lambda_I + \lambda_E}$. When $\rho \in \left(\frac{\bar{\rho}}{\lambda_I + \lambda_E}, \frac{\bar{\rho}(1-\lambda_I)}{\lambda_E}\right)$ the patentee's expected payoff is

$$\pi_P = [\lambda_E(1 - a^*) + \lambda_I(1 - a^*)(1 - \phi^*)]\rho\beta + \lambda_I\phi^*\pi_P(\rho).$$

Using that $\lambda_I = \lambda - \lambda_E$, that a^* is independent of λ_E, we have

$$\frac{\partial \pi_P}{\partial \lambda_E} = \left[\lambda_I \frac{\partial \phi^*}{\partial \lambda_E} - \phi^*\right][\pi_P(\rho) - (1 - a^*)\rho\beta]$$

From the definition of ϕ^* we find that $\lambda_I \frac{\partial \phi^*}{\partial \lambda_E} = \phi^*$, so the payoff is independent of λ_E. Finally, when $\rho > \frac{\bar{\rho}(1-\lambda_I)}{\lambda_E}$ the patentee gets zero. Since for any other value of ρ the patentee gets a strictly positive expected payoff, the patentee is better off whenever λ_E is as small as possible, to shink the region $\rho > \frac{\bar{\rho}(1-\lambda_I)}{\lambda_E}$. □

A.1 Proposition 7: Fixed Cost and Uncertain Success

Proof. **Approach.** If the patentee approaches, the developer can pay a fee T to use the patentee's knowledge/patent to implement B and succeed with probability 1 without the need to pay the cost K. In that case, the patentee gets T and the developer gets $\pi - T$. If the deal is rejected, the developer will hold a belief $\hat{\lambda} = 1$ and not invest to develop B if $\bar{\rho} < \alpha\rho + K + 1 - \alpha$. In that case, payoffs are 0 for the patentee and $(1 - \bar{\rho})\pi$ for the developer. If $\bar{\rho} \geq \alpha\rho + K + 1 - \alpha$, the developer will invest to develop B and payoffs are $\alpha\rho\pi$ for the patentee and $(\alpha(1 - \rho) - K)\pi$ for the developer.

Case 1. If $\bar{\rho} < \alpha\rho + K + 1 - \alpha$, the joint payoff from agreement is π while the disagreement payoff is $(1 - \bar{\rho})\pi$. Thus, the equilibrium transfer is $T = \beta\bar{\rho}\pi$. (This is the same as in the baseline case). The patentee extract rents for *encouraging efficient adoption*.

Case 2. If $\bar{\rho} \geq \alpha\rho + K + 1 - \alpha$, the joint payoff from agreement is π. Joint disagreement payoffs are $(\alpha - K)\pi$. Thus, the equilibrium transfer is $T = \alpha\rho\pi + \beta(1 - \alpha + K)\pi$. The baseline case obtains when $K = 0$. In contrast to the baseline case, the patentee is able to extract rents from his "know-how," which *avoid costly duplication effort*.

Lemma 2. *If the patentee approaches ex ante, the Nash-barganing payoffs are*

$$\pi_P(\rho) = \begin{cases} \beta\bar{\rho}\pi & \text{if } \bar{\rho} < \alpha\rho + K + 1 - \alpha \\ (\alpha\rho + \beta(1 - \alpha + K))\pi & \text{if } \bar{\rho} \geq \alpha\rho + K + 1 - \alpha \end{cases}$$

$$\pi_D(\rho) = \begin{cases} (1 - \beta\bar{\rho})\pi & \text{if } \bar{\rho} < \alpha\rho + K + 1 - \alpha \\ (1 - \alpha\rho - \beta(1 - \alpha + K))\pi & \text{if } \bar{\rho} \geq \alpha\rho + K + 1 - \alpha \end{cases}.$$

Equilibrium Analysis:

1. $\phi^* = 1$. In this case, $\hat{\lambda} = 0$, so (8) becomes $\bar{\rho} \geq K + 1 - \alpha$.
   (a) If $\bar{\rho} \geq K + 1 - \alpha$, the developer invests to adopt B if not approached.
      i. If $\bar{\rho} < \alpha\rho + K + 1 - \alpha$, the patentee gets $\alpha\rho\pi$ by not approaching and $\beta\bar{\rho}\pi$ by approaching. Approaching reveals the existence of the patentee which changes the decision to invest when the negotiation breaks down. Thus, approaching is optimal when $\alpha\rho \leq \beta\bar{\rho}$. This equilibrium exists when

$$\rho \in \left(\frac{\bar{\rho} - K - 1 + \alpha}{\alpha}, \frac{\beta\bar{\rho}}{\alpha}\right].$$

ii. If $\bar{\rho} \geq \alpha\rho + K + 1 - \alpha$ it is optimal to approach to capture rents from avoiding costly duplication effort and also to prevent failure in the adoption, which happens with probability $1 - \alpha$. The additional surplus of approaching is $\beta(1 - \alpha + K)\pi$.

(b) If $\bar{\rho} < K + 1 - \alpha$., the developer never invests to adopt B if not approached. It is optimal to approach because the patentee gets 0 by not approaching.

2. $\phi^* = 0$. In this case, $\hat{\lambda} = \lambda$, so (8) becomes $\frac{\bar{\rho}-K-1+\alpha}{\alpha\lambda} \geq \rho$.

(a) When $\frac{\bar{\rho}-K-1+\alpha}{\alpha\lambda} \geq \rho$, the developer invests to develop B if not approached.

i. If $\bar{\rho} < \alpha\rho + K + 1 - \alpha$, the patentee gets $\alpha\rho\pi$ by not approaching and $\beta\bar{\rho}\pi$ by approaching. Approaching reveals the existence of the patentee which changes the decision to invest when the negotiation breaks down. Thus, not approaching is optimal when $\alpha\rho \geq \beta\bar{\rho}$. This equilibrium exists when

$$\rho \in \left(\max\left\{\frac{\bar{\rho} - K - 1 + \alpha}{\alpha}, \frac{\beta\bar{\rho}}{\alpha}\right\}, \frac{\bar{\rho} - K - 1 + \alpha}{\alpha\lambda}\right].$$

ii. If $\bar{\rho} \geq \alpha\rho + K + 1 - \alpha$ it is optimal to approach to capture rents from avoiding costly duplication effort and preventing failure to discover B. Thus, there is no equilibrium with $\phi^* = 0$ in this region.

(b) When $\frac{\bar{\rho}-K-1+\alpha}{\alpha\lambda} < \rho$, the developer never invests in developing B if not approached. It is optimal to approach because the patentee gets 0 by not approaching, so an equilibrium where the patentee never approaches is not an equilibrium in this region.

3. $\phi^* \in (0, 1)$. Suppose the developer is indifferent between A and invest to develop B, and chooses A with probability a. For the developer to be indifferent we need the patentee to approach with probability ϕ such that

$$\bar{\rho} - \alpha\hat{\lambda}(\phi)\rho = K + 1 - \alpha.$$

Since $\hat{\lambda}(\phi) \in [0, \lambda]$, we need $\bar{\rho} \geq K + 1 - \alpha$.It is possible to meet the inequality above when

$$\frac{\bar{\rho} - K - 1 + \alpha}{\alpha\lambda} \leq \rho.$$

Note that this condition implies $\bar{\rho} < \alpha\rho + K + 1 - \alpha$ meaning that the developer chooses A if approached but the negotiation breaks down. Thus, the patentee gets $\beta\bar{\rho}\pi$ by approaching. By not approaching, the developer invests in B with probability

$(1-a)$, so the patentee gets $(1-a)\alpha\rho\pi$. Thus, the patentee is indifferent between approaching and not when

$$(1-a)\alpha\rho = \beta\bar{\rho}.$$

It is possible to find a to satisfy this condition when $\rho \geq \frac{\beta\bar{\rho}}{\alpha}$. Thus, there exists a mixed equilibrium when

$$\rho \geq \max\left\{\frac{\beta\bar{\rho}}{\alpha}, \frac{\bar{\rho}-K-1+\alpha}{\alpha\lambda}\right\}$$

If the mixed equilibrium exists, the mixing probabilities are

$$a^* = 1 - \frac{\beta\bar{\rho}}{\alpha\rho}, \quad \phi^* = 1 - \frac{(\bar{\rho}-K-1+\alpha)(1-\lambda)}{\lambda(1-\alpha+\alpha\rho-\bar{\rho}+K)},$$

□

Proof of the Proposition 8

Proof. Consider a developer who holds a belief $\hat{\lambda}$ conditional on not being approached. Let $V_A = (1-\bar{\rho})\pi$ and $V_B = (1-\hat{\lambda}\rho)\pi$. The developer's expected payoff from attempting to innovate with probability α is

$$V_A + \alpha(V_B - V_A) - c(\alpha).$$

Lemma 3. *Conditional on a belief $\hat{\lambda}$, the developer will invest in technology B if and only if $\bar{\rho} > \hat{\lambda}\rho$. His optimal choice, $\alpha^*(\hat{\lambda})$, is given by*

$$\alpha^*(\hat{\lambda}) = \begin{cases} 1 & \text{if } \bar{\rho} - \hat{\lambda}\rho \geq k \\ \frac{\bar{\rho}-\hat{\lambda}\rho}{k} & \text{if } 0 < \bar{\rho} - \hat{\lambda}\rho < k \\ 0 & \text{if } \bar{\rho} - \hat{\lambda}\rho \leq 0. \end{cases} \tag{9}$$

Note that $\alpha^*(0)$ is positive in all cases. Also, we have $\alpha^*(\lambda) \leq \alpha^*(0)$, and this is strict iff $\alpha^*(\lambda) < 1$. Also, $\alpha^*(\hat{\lambda}(\phi))$ is clearly continuous in ϕ (and all other parameters). Moreover, $\alpha^*(\hat{\lambda}(\cdot))$ is strictly increasing at every ϕ_0 such that $\alpha^*(\hat{\lambda}(\phi_0))$ is interior (otherwise $\alpha^*(\hat{\lambda}(\cdot))$ is locally flat). We characterize the equilibria in this game in the proposition below.

First, we show that the developer never mixes in equilibrium. Note that he will invest a positive amount iff $\bar{\rho} > \hat{\lambda}\rho$. However, this inequality ensures that, if he is successful, then he will strictly prefer to adopt technology B at the margin (after his investment cost is sunk). One can easily verify that this inequality also implies that his expected overall payoff (including the investment cost) from investing $\alpha^*(\hat{\lambda})$ is strictly larger than his payoff from picking A for sure. Thus, he will never mix when $\alpha^*(\hat{\lambda}) > 0$. And we can never have $\alpha^*(\hat{\lambda}) = 0$ in an equilibrium (for the same reason that we can never have the developer pick A for sure in an equilibrium of the baseline game). Turning to the patentee's equilibrium conduct, there are a few cases to consider.

1. Suppse $\rho \leq \bar{\rho}$. Then the developer's outside option is to use B, so the patentee gets $\rho\pi$ if he approaches and $\alpha^*(\lambda)\rho\pi$ if he waits. Thus, there definitely exists an equilibrium with $\phi^* = 1$. However, if there exists any $\phi_0 < 1$ such that $\alpha^*(\hat{\lambda}(\phi_0)) = 1$, then $\phi^* = \phi_0$ is also an equilibrium.

2. Suppose that $\rho > \bar{\rho}$ and $\alpha^*(\lambda) = 1$ ($\Rightarrow \alpha^*(0) = 1$). Note that this implies $\bar{\rho} > \lambda\rho$ (otherwise we'd have $\alpha^*(\lambda) = 0$). Thus, we must have $\rho \in (\bar{\rho}, \bar{\rho}/\lambda)$. Note that in this case we have $\alpha^*(\hat{\lambda}(\phi)) = 1$ for any $\phi \in [0, 1]$. Thus, the equilibrium decision-making is exactly the same as the baseline model with $\rho \in (\bar{\rho}, \bar{\rho}/\lambda)$. Hence, as in the baseline game, the unique equilibrium is $\phi^* = 0$. However, the developer's payoff smaller than in the baseline game, due to the investment cost $c(\alpha^*)$.

3. Suppose that $\rho > \bar{\rho}$ and $\alpha^*(\lambda) < 1$ ($\Rightarrow \alpha^*(\lambda) < \alpha^*(0)$). Conditional on a belief $\hat{\lambda}$, the patentee gets $\alpha^*(\hat{\lambda})\rho\pi$ if he waits, whereas he gets $\beta\bar{\rho}\pi$ if he approaches. There are a few subcases.

   (a) Suppose that $\alpha^*(\lambda)\rho < \alpha^*(0)\rho \leq \beta\bar{\rho}$. Then there is a unique equilibrium with $\phi^* = 1$, because the highest attainable success probability ($\alpha = \alpha^*(0)$) is still not high enough to make waiting more profitable than approaching.

   (b) Suppose that $\alpha^*(\lambda)\rho < \beta\bar{\rho} < \alpha^*(0)\rho$. Then $\phi = 1$ cannot be an equilibrium, because when the developers belief is $\hat{\lambda} = 0$ the patentee would strictly prefer to deviate to waiting. However, there necessarily exists an equilibrium with $\phi^* \in (0, 1)$. This is the value of ϕ^* that satisfies $\alpha^*(\hat{\lambda}(\phi^*))\rho = \beta\bar{\rho}$. By the intermediate value theorem, there exists a solution to this equation. And it must be unique because $\alpha^*(\hat{\lambda}(\phi))$ is strictly increasing in ϕ whenever $\alpha^*(\hat{\lambda}(\phi))$ is interior (which it must be to satisfy the equation, given that $\alpha^*(0)\rho > \beta\bar{\rho} > \alpha^*(\lambda)\rho$).

   (c) Suppose that $\beta\bar{\rho} \leq \alpha^*(\lambda)\rho < \alpha^*(0)\rho$. Then waiting is strictly better than approaching even when α^* takes its lowest possible value ($\alpha^* = \alpha^*(\lambda)$). Thus, there is a unique equilibrium with $\phi^* = 0$.

Equilibrium analysis as a function of the exogenous parameters.

1. $\phi^* = 1$; The developer approaches with probability 1.

   (a) When $\rho \leq \bar{\rho}$ this is always an equilibrium because by not approaching the developer gets $\alpha^* \rho\pi$, whereas approaching gives the patentee $\rho\pi$.

   (b) When $\rho > \bar{\rho}$ the patentee gets $\beta\bar{\rho}\pi$ from not approaching, and the developer in equilibrium must hold the belief $\hat{\lambda} = 0$ and, therefore, $\alpha^* = \min\left\{\frac{\bar{\rho}}{k}, 1\right\}$. This is an equilibrium when $\alpha^*\rho \leq \beta\bar{\rho}$, and not an equilibrium otherwise.

      i. When $k \geq \bar{\rho}$, $\alpha^* \leq 1$, so for this equilibrium to exists we need $\rho \leq \beta k$. Thus, the equilibrium exists whenever $\bar{\rho} < \rho \leq \beta k$.

      ii. When $k < \bar{\rho}$, this equilibrium does not exists because we cannot have $\bar{\rho} < \rho \leq \beta\bar{\rho}$.

2. $\phi^* = 0$. The developer approaches with probability 0 so we must have $\hat{\lambda} = \lambda$.

   (a) Consider first the case $\rho \leq \bar{\rho}$. If $\alpha^* = 1$, which occurs when $\bar{\rho} \geq \lambda\rho + k$, then the patentee is indifferent between approaching or not, so this is an equilibrium. If $\alpha^* < 1$, which occurs when $\bar{\rho} < \lambda\rho + k$, this is not an equilibrium because the patentee gets $\alpha^*\rho\pi < \rho\pi$, so it has an incentive to approach.

   (b) Consider next the case $\rho > \bar{\rho}$. If $\alpha^* = 1$, which occurs when $\bar{\rho} \geq \lambda\rho + k$, this is an equilibrium, because $\rho > \beta\bar{\rho}$. If $\alpha^* < 1$, which occurs when $\bar{\rho} < \lambda\rho + k$, then this is an equilibrium as long as

$$\left(\frac{\bar{\rho} - \lambda\rho}{k}\right)\rho \geq \beta\bar{\rho} \Leftrightarrow \lambda\rho^2 - \bar{\rho}\rho + \beta k\bar{\rho} \leq 0$$

   This condition is equivalent to

$$\rho \in [\rho^-, \rho^+] \equiv \left[\frac{1}{2\lambda}\left(\bar{\rho} - \sqrt{\bar{\rho}^2 - 4\lambda\beta k\bar{\rho}}\right), \frac{1}{2\lambda}\left(\bar{\rho} + \sqrt{\bar{\rho}^2 - 4\lambda\beta k\bar{\rho}}\right)\right].$$

   In some cases, $\rho^- < \bar{\rho}$, so this will be an equilibrium in $(\bar{\rho}, \rho^+]$.[17] This is not an equilibrium if $\rho^+ < \bar{\rho}$. This is the case when

$$\sqrt{\bar{\rho}^2 - 4\lambda\beta k\bar{\rho}} < (2\lambda - 1)\bar{\rho}$$

[17] For example, when $\beta k < \bar{\rho}(1 - \lambda)$.

If $\lambda \leq 0.5$ the condition above never holds, meaning that this equilibrium exists. If $\lambda > 0.5$ and $(1-\lambda)\bar{\rho} < \beta k$, the condition above holds, meaning that this equilibrium does not exist.

3. $\phi^* \in (0,1)$. The patentee mixes between approaching and waiting.

   (a) First, suppose that $\bar{\rho} < k$, so $\alpha^* < 1$. We cannot have an equilibrium for $\rho \leq \bar{\rho}$ because by approaching the patentee gets $\rho\pi$ and by waiting the patentee gets $\alpha^*\rho\pi$. For $\rho > \bar{\rho}$ for an equilibrium to exists we need

   $$\left(\frac{\bar{\rho} - \hat{\lambda}\rho}{k}\right)\rho = \beta\bar{\rho}.$$

   In contrast to the baseline case, it may not be possible to sufficiently lower the payoff of not approaching to achieve this equality. The lowest the payoff from waiting can be is when $\hat{\lambda} = \lambda$, and $\alpha^* = \frac{\bar{\rho}-\lambda\rho}{k}$. A necessary condition for this equilibrium to exist is

   $$\left(\frac{\bar{\rho} - \lambda\rho}{k}\right)\rho > \beta\bar{\rho},$$

   which is the same as $\rho \notin [\rho^-, \rho^+]$. We also need $\rho > \beta k$ because mixing can only decrease α^* from $\frac{\bar{\rho}}{k}$. If these conditions hold, we can then find $\phi \in (0,1)$ such that the belief derived from Bayesian updating, $\lambda(\phi)$, satisfies the equality $\left(\frac{\bar{\rho}-\lambda(\phi)\rho}{k}\right)\rho = \beta\bar{\rho}$.

   (b) Next, suppose that $\bar{\rho} \geq k$, so $\alpha^* = 1$ when $\hat{\lambda} = 0$.

      i. If $\alpha^* = 1$ when $\hat{\lambda} = \lambda$, which happens when $\bar{\rho} \geq \lambda\rho + k$, then for any $\hat{\lambda} \in [0,\lambda]$ we have $\alpha^* = 1$. If $\rho \leq \bar{\rho}$ the patentee is indifferent so this is an equilibrium. If $\rho > \bar{\rho}$, for an equilibrium to exists we need $\rho = \beta\bar{\rho}$, which cannot happen.

      ii. If $\alpha^* < 1$ when $\hat{\lambda} = \lambda$, which happens when $\bar{\rho} < \lambda\rho + k$, then for any $\hat{\lambda} \in [0,\lambda]$ we have $\alpha^* = 1$. If $\rho \leq \bar{\rho}$ the strictly prefers to approach, so this is not an equilibrium. If $\rho > \bar{\rho}$, for an equilibrium to exists we need

      $$\left(\frac{\bar{\rho} - \lambda\rho}{k}\right)\rho \leq \beta\bar{\rho},$$

      in which case we can find $\hat{\lambda}(\phi)$ to satisfy the equality $\left(\frac{\bar{\rho}-\lambda(\phi)\rho}{k}\right)\rho = \beta\bar{\rho}$.

□